Quantitative Data Analysis: A Companion for Accounting and Information Systems Research

Teaching Materials

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1. Introduction
2. Comparing Differences Across Groups
3. Assessing (Innocuous) Relationships
5. Nested Data and Multilevel Models: Hierarchical Linear Modeling
6. Analyzing Longitudinal and Panel Data
7. Causality: Endogeneity Biases and Possible Remedies
8. How to Start Analyzing, Test Assumptions and Deal with that Pesky p-Value
9. Keeping Track and Staying Sane

What these materials are about
Offering a guide through the essential steps required in quantitative data analysis
Part 5: Hierarchical Linear Modeling
1. Theoretical background
   - Multilevel research
   - Assumptions, agreement and reliability
   - Building the measurement model
   - (Multilevel) regressions and Random Coefficient Modeling

2. Demonstration using HLM7
   - Importing data
   - Specifying the model
   - Interpreting results
THEORETICAL BACKGROUND
Multilevel research: what is it?

- Effect of group membership on individual outcomes
  --> Top-down

- E.g. effect of IQ (level 1), teacher expectations (level 2) on student performance
Multilevel research: what is it?

Level 2
- Class 1

Level 1
- Student 1
- Student 2
- Student x

Level 2
- Class 2

Level 1
- Student 1
- Student 2
- Student y

Within-group

Between-group

School 1

Teacher expectations

Student IQ

Student performance
Multilevel research: traditional approaches

- Aggregate individual-level variables
  - Individual variance is lost (IQ)
  - Effects get inflated because of the loss of variance

- Assign group-level variables to individuals
  - Violation of ‘independence of observations’ (correlated error terms)

=> Hierarchical Linear Modeling
Assumptions, agreement and reliability (1)

- Assumptions that apply for regression
  - (Reliable and valid measurement model)
  - Linearity
  - Normality
  - Independence
  - Homoscedasticity (homogeneity of variance)

- However:
  - Independence and homoscedasticity only within-group
  - Additional assumptions when using peer-ratings (e.g. students rate teacher)
    a. Inter-rater consistency
    b. Reliability of group means
    c. Inter-rater agreement
a. Inter-rater consistency: Intraclass Correlation (1) (ICC(1))

- ICC(1) = $\frac{\tau_{00}}{\tau_{00} + \sigma^2}$
  - $\tau_{00}$ = between-group variance
  - $\sigma^2$ = within-group variance
    - Part of variance that is explained by between-group variance (i.e. by group membership)

- From ANOVA: ICC(1) = $\frac{MSB - MSW}{MSB + [(k-1) \times MSW]}$
  - MSB = between-group mean square
  - MSW = within-group mean square
  - k = within-group size

(Bliese, 2000)
b. Reliability of group means: Intraclass Correlation (2) (ICC(2))

- From ANOVA: $\text{ICC}(2) = \frac{\text{MSB} - \text{MSW}}{\text{MSB}}$
  - MSB = between-group mean square
  - MSW = within-group mean square
    - Part of between group variance that is not explained by within-group variance

(Bliese, 2000)
c. Within-group inter-rater agreement: $r_{wg}^*$

\[ r_{wg}^* = 1 - \frac{\sigma_x^2}{\sigma_{EU}^2} \]

- $\sigma_x^2$ = average within-group variance
- $\sigma_{EU}^2$ = variance under uniform distribution = \((A^2 - 1)/12\)

- Ratio of average within-group variance per estimated variance without groups

(Lindell, Brandt, & Whitney, 1999)
Recap: Homoscedasticity / Homogeneity of Variance

- Two variables (e.g. regression): spread of errors/residuals is equal across different values of $x$
In many statistical tests
- Sampling distribution is normally distributed
  --> test normality of sample
- Visually testing normality of (sub-)sample data
  - Histograms (see slide 10)
  - Q-Q plots: theoretical vs. actual quantiles

Recap: Normality

Skew (+)  Kurtosis (+)
Recap: Normality

- Statistical tests for normality of (sub-)sample data
  - Compute descriptives including skew and kurtosis
  - Convert skew and kurtosis to z-scores, e.g.:
    \[
    z_{\text{skewness}} = \frac{\text{skewness} - \mu}{\text{SE}_{\text{skewness}}} \Rightarrow \frac{|\text{skewness}|}{\text{SE}_{\text{skewness}}} \text{ must be} \leq 1.96
    \]
    - Increase to 2.58 in larger samples and do not use in very large samples \((n > 200)\)

- Shapiro-Wilk test: significant \((p < .05)\) when NOT normal
Recap: Normality

- In regression-based models
  - Errors/residuals, not indicators need to be normally distributed
  - Same visual principles as Q-Q plot apply

Please note: in this case, both graphs do not represent the same data.
What if assumptions are violated?

- Correct data
  - Exclude outliers
  - Transform data, e.g.:
    - Log-, square root and reciprocal (1/x) transformations shorten the right tale (i.e. correct positive skew)
    - The same transformations applied to the reverse score (score – highest score + 1) correct for negative skew

⚠️ The same transformation has to be applied to variables that are compared directly

- Turn to tests that are robust against violations or to non-parametric tests, e.g.
  - Mann–Whitney U for group comparisons
  - Kendall's tau for dependence between two variables
Building the measurement model

- For individual-level variables (IQ)
  - Classical approach to creating and evaluating scales (e.g., CFA, Cronbach’s Alpha)

- For group-level variables (teacher expectations)
  - Test for consistency, reliability of group means, agreement based on individual-level items
  - Aggregate items to the group level (average)
  - Create and evaluate scale based on aggregated items


(Peterson and Castro, 2006)
(Multilevel) regressions

- Classical linear regression

\[ Y_i = \beta_0 + \beta_1 X_i + r_i \]
(Multilevel) regressions

- Hierarchical Linear Model
  - Level 1: \( Y_i = \beta_{0j} + \beta_{1j}X_{ij} + r_i \)
  - Level 2: \( \beta_{0j} = \gamma_{00} + \gamma_{01}Z_j + \mu_{0j} \)
    \( \beta_{1j} = \gamma_{10} + \gamma_{11}Z_j + \mu_{1j} \)
(Multilevel) regression / HLM / Random Coefficient Modeling

Cross-sectional

Longitudinal
Different types of models

- Hierarchical linear models
  - 2 Levels
  - More levels
  - E.g. student performance per class (per school)

- Hierarchical multivariate linear models
  - 2 or more levels
  - Multiple outcome variables
  - E.g. student performance per class over time

- Cross-classified multilevel models (see Leckie 2013)
  - 2 or more levels
  - Units can belong to multiple groups
  - E.g. student performance per class and neighbourhood
APPLICATION

Hierarchical Linear and Nonlinear Modeling
Import data (the easy way)

- Make level 1 and level 2 file
  - Level 1: each student is one row (case)
  - Level 2: each class/teacher is one row (case)
Recap: Structuring data

- One row per case, one variable per column

<table>
<thead>
<tr>
<th>Student</th>
<th>Age</th>
<th>Class</th>
<th>IQ</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19</td>
<td>1</td>
<td>95</td>
<td>5.9</td>
</tr>
<tr>
<td>2</td>
<td>53</td>
<td>1</td>
<td>93</td>
<td>6.3</td>
</tr>
<tr>
<td>3</td>
<td>27</td>
<td>2</td>
<td>105</td>
<td>6.5</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>107</td>
<td></td>
<td>4.7</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

- Depends on unit of analysis (e.g. person)
### Structuring data

- Multilevel data: split into level 1 and level 2

#### Level 1

<table>
<thead>
<tr>
<th>Student</th>
<th>Class</th>
<th>IQ</th>
<th>Performance</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>95</td>
<td>5.9</td>
<td>...</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>93</td>
<td>6.3</td>
<td>...</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>105</td>
<td>6.5</td>
<td>...</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>107</td>
<td>4.7</td>
<td>...</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>118</td>
<td>5.4</td>
<td>...</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
<td>79</td>
<td>5.5</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

#### Level 2

<table>
<thead>
<tr>
<th>Class</th>
<th>Expectation</th>
<th>...</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
<td>...</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>...</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>...</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>
- Make level 1 and level 2 file
  - Level 1: each student is one row (case)
  - Level 2: each class/teacher is one row (case)

- Build Multivariate Data Matrix (MDM)
Import data (the easy way)
Specifying the model

LEVEL 1 MODEL (bold: group-mean centring; bold italic: grand-mean centring)
PERFORMA = \( F_0 + r \)

LEVEL 2 MODEL (bold italic: grand-mean centring)
\( F_0 = \beta_0 + u_0 \)
Mean centring

- RAW: original score
- Group: individual score minus group mean
  - Does not control for between-group variance in level 1 variables when testing level 2 variables
- Grand: individual score minus total sample mean
  - Yields intercepts and slope parameters that are easier to interpret
    - Overall average becomes reference point
    - E.g. if I have average intelligence, my group membership will have $\gamma_{00}$ influence on my performance
Interpreting the results

- Based on basic model = one-way ANOVA
  - “reliability estimate” = ICC(1)
  - “final estimation of variance component” tests significance of between-group variance

\[
\sigma^2 = 1.33760 \\
\gamma_0 = 0.11949 \\
\gamma_1 = 0.260
\]

- Robust SE means robust against violations of assumptions --> if non-robust SE and robust SE differ you should check assumptions
  - Residual plots like with regression
  - Other assumptions: see Raudenbush and Bryk (2002)

- Coefficient for intercept level 2 (\(\gamma_{00}\)) = average performance
Interpreting the results

- More advanced model

  - Coefficient is similar to regression
  - Coefficient is different from 0 when $t$-ratio is significant
Test for multilevel mediation

- (Unrealistic) example: teacher expectations $\rightarrow$ IQ $\rightarrow$ student performance
  - Step 1: level 1 model: $\text{PERF}_{ij} = \beta_{0j} + r_{ij}$
    level 2 model: $\beta_{0j} = \gamma_{00} + \gamma_{01} \cdot \text{EXPECT}_{ij} + u_{0j}$
  - Step 2: level 1 model: $\text{PERF}_{ij} = \beta_{0j} + \beta_{1j} \cdot \text{IQ}_{ij} + r_{ij}$
    level 2 model: $\beta_{0j} = \gamma_{00} + \gamma_{01} \cdot \text{EXPECT}_{ij} + u_{0j}$
  - Step 3: level 1 model: $\text{IQ}_{ij} = \beta_{0j} + r_{ij}$
    level 2 model: $\beta_{0j} = \gamma_{00} + \gamma_{01} \cdot \text{EXPECT}_{ij} + u_{0j}$
  - Step 4: Interpret coefficients
    - Coefficient of EXPECT not significant in Step 3? No mediation
    - Coefficient of EXPECT equal in step 1 and 2? No mediation
    - Coefficient of EXPECT significant in Step 1 and step 3, but not in step 2? Full mediation
    - Coefficient of EXPECT significant in all steps, but lower in step 2 than 1? Partial mediation

- If full mediation: Level 1: $\text{PERF}_{ij} = \beta_{0j} + \beta_{1j} \cdot \text{IQ}_{ij} + r_{ij}$
  Level 2: $\beta_{0j} = \gamma_{00} + u_{0j}$
  $\beta_{1j} = \gamma_{10} + \gamma_{11} \cdot \text{EXPECT}_{j} + u_{1j}$

Krull and MacKinnon (1999, 2001)
Referred to in slides


Key resources


- Software: http://www.ssicentral.com/hlm/resources.html

End of Part 5