Accounting and revenue management (WE-30)

2 - Simulating Fare Class Choice Behaviour with Flexible Substitution Patterns in Airline Revenue Management

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Accounting for individual choice decisions, especially regarding buy-up and buy-down behaviour of customers in airline revenue management, is both a complex stochastic and dynamic problem. Hence, dynamic optimization is able to provide an exact solution. In terms of capacity control, in airline revenue management an application of the before mentioned method has proven to be rather inappropriate due to high computational and memory requirements. As a consequence, approximate methods are usually applied instead. Our aim is to provide an alternative approach based on the simulation of a flexible number of individuals with associated utility values and improved revenue performance. Thus, we are able to incorporate individual decision making behaviour regarding the choice of fare classes (alternatives). Utility values are obtained from random utility models (RUM) like the Multinomial Logit model (MNL) which is popular for its Independence from Irrelevant Alternatives (IIA) assumption resulting in constant substitution patterns. However, in situations where the choice set contains alternatives that share common unobserved attributes more flexible choice models like the Nested Logit model (NL) or Mixed Logit model (MMNL) are to be favored. These models exhibit flexible substitution patterns and, thus, are able to capture shifts in demand between alternatives that share common characteristics not observed by the modeller. In our analysis we take advantage of this characteristic to model buy-up and buy-down behaviour between fare classes. Consequently, by implementing simulation procedures in a mathematical model, we are finally able to provide a comparison of revenue development under both stochastically independent and dependent demand structure.
Simulating Fare Class Choice Behaviour with Flexible Substitution Patterns in Airline Revenue Management

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Introduction
Starting Point

- Static models of quantity based Revenue Management
  - For \( f \geq 2 \) fare classes
  - Ordered demand over time
  - Resource with capacity \( C \)

Objective

- Revenue maximization through an optimal allocation of capacity to different products
Approach

Motivation

- Simplifying assumptions for static single-resource models
  - Demand for different booking classes are stochastically independent
  - Demand does not depend on other available booking classes
- Constant substitution patterns for modeling buy-up/buy-down
  - Customer preference orders/Multinomial Logit-Model

Intention

- Relaxation of simplifying assumptions
- Consideration of stochastically dependent demand structures

Realization

- **Appropriate** Discrete Choice Models
- **Simulation** based model for revenue optimization
Part I: Customer Choice
Preface

- Modeling of fare class choice behaviour
  - Discrete choice models
- Distinction in models with:
  1. Constant substitution patterns → MNL
  2. Flexible substitution patterns → NL
- Generation of synthetic data for both MNL and NL
Part I: Customer Choice

Dataset Generation I

MNL

\[ U_i = V_i + \varepsilon_i \quad \forall \ i \]

- Deterministic part:
  \[ V_i = \sum_h \beta_{ih} x_{ih} \]
  → linear-in-parameters
- Stochastic part: \( \varepsilon_i \)
  → iid EV distributed
- Constant substitution patterns

NL

\[ U_i = V_i + [\varepsilon_{mi} + \varepsilon_m] \quad \forall \ m, i \]

- Alternatives with common unobserved attributes are grouped into \( m \) nests
- Deterministic part: \( V_i \)
- Stochastic part: \( \varepsilon_{mi} + \varepsilon_m \)
- Flexible substitution patterns
Part I: Customer Choice

Dataset Generation II

- Choice set contains 5 fare class alternatives
  - Regular (1) and discount (2) fare in Business Class
  - Regular (3) and discount (4) fare in Economy Class
  - No choice/Outside Alternative (OA) (5)

- Decision variables, reflect price sensitive individual
  - Price = alternative-specific, normal distributed random variables
  - Flexibility = categorial variable
  - Trip purpose = dummy variable (Business, Leisure)
  - Sex = dummy variable (Male, Female)

Considered function of deterministic utility

\[ V_i = \beta_{asci} + \beta_1 \cdot x_{price,i} + \beta_2 \cdot x_{flex,i} + \beta_3 i \cdot x_{prp} + \beta_4 i \cdot x_{sex} \quad \forall \ i \]
Substitution Patterns I

MNL

<table>
<thead>
<tr>
<th>Fare class</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market share</td>
<td>5.10</td>
<td>10.50</td>
<td>45.30</td>
<td>31.00</td>
<td>8.10</td>
</tr>
</tbody>
</table>

- Market shares when all fare classes are available

<table>
<thead>
<tr>
<th>Fare class</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market share</td>
<td>6.96</td>
<td>14.97</td>
<td>65.56</td>
<td>0.00</td>
<td>12.51</td>
</tr>
</tbody>
</table>

- Alternative 4 is no longer available
- Market shares of 1, 2, 3 and 5 rise proportionally
- Ratio of substitution between any pair of alternatives is constant

→ **Example:** ratio of alternatives \[ \frac{1}{3} = \frac{5.10}{45.30} = \frac{6.96}{65.56} = 0.11 \]
Substitution Patterns II

NL

<table>
<thead>
<tr>
<th>Fare class</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market share</td>
<td>2.93</td>
<td>6.67</td>
<td>40.41</td>
<td>28.29</td>
<td>21.70</td>
</tr>
</tbody>
</table>

- Nest 1 = \{1,2\}, Nest 2 =\{3,4\}, Nest 3 =\{5\}

<table>
<thead>
<tr>
<th>Fare class</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market share</td>
<td>3.53</td>
<td>8.26</td>
<td>60.03</td>
<td>0.00</td>
<td>28.18</td>
</tr>
</tbody>
</table>

- Market share of alternative in the same nest (=3) rises proportionally
- Flexible substitution across nests, constant substitution within nests

→ **Example:** ratio of alternatives \( \frac{1}{3} = \frac{2.93}{40.41} = 0.073 \neq \frac{3.53}{60.03} = 0.059 \)

→ **Example:** ratio of alternatives \( \frac{1}{2} = \frac{2.93}{6.67} = \frac{3.53}{8.26} = 0.43 \)
Summary - Customer Choice

Application of NL - Models:

- Allows for dependencies in demand structure
- Reflects more realistic individual choice behaviour than MNL
- More precise predictions regarding buy-up/buy-down

⇒ Data generation process is now applied in a mathematical model
⇒ Obtained Revenue from NL is assumed to represent the true value
Part II: Simulation Study
Preface

Simulation Model

▶ **Idea:** Simulation of a population of individuals
   → With affiliated utility values from NL (and MNL for comparison)
   → Highest utility value determines choice decision
   → Differences in revenue for dependent and independent demand structure

▶ No non-linearities → utility values instead of choice probabilities
▶ Solvable with Cplex in GAMS version 23.5.2
Part II: Simulation Study

Definitions

Sets

\[ f \]
fare class, no choice/OA \( f = 5 \)

\[ n \]
simulated number of individuals

Parameters

\[ p_f \]
price of fare \( f \)

\[ u_{n,f} \]
normalized utility \((0 \leq u_{n,f} \leq 1)\) of individual \( n \) choosing \( f \)

\[ C \]
resource capacity
Part II: Simulation Study

Definitions

Variables

\( b_f \) \hspace{1cm} \text{actual number of bookings in } f \ (\text{booking limit})

\( y_{n,f} \) \hspace{1cm} = 1, \text{ if individual } n \text{ chooses } f \ (0, \text{ else})

\( x_{n,f} \) \hspace{1cm} = 1, \text{ if } f \text{ is offered to } n \ (0, \text{ else})
Part II: Simulation Study

Model I

\[
\begin{align*}
\max F &= \sum_{n,f} p_f y_{n,f} \\
\quad &\quad \text{for all } n, f, f' \\
\quad &\quad \text{for all } f, f < 5 \\
\quad &\quad \text{for all } n, f < 5 \\
\quad &\quad \text{for all } n \\
\quad &\quad \text{for all } n, f < 5 \\
\end{align*}
\]
Part II: Simulation Study

Model II

\[ y_{n,f} - x_{n,f} \leq 0 \quad \forall \ n, f \] (7)
\[ x_{n,f} - x_{n-1,f} \leq 0 \quad \forall \ n > 1, f < 5 \] (8)
\[ b_f \geq 0 \quad \forall \ f \] (9)
\[ b_f - C \leq 0 \quad \forall \ f \] (10)
\[ x_{n,f} \in \{0, 1\} \quad \forall \ n, f \] (11)
\[ y_{n,f} \in \{0, 1\} \quad \forall \ n, f \] (12)
Results
## High Demand Load Factor

<table>
<thead>
<tr>
<th></th>
<th>MNL</th>
<th>NL_1(^1)</th>
<th>NL_2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>Customers</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>bookings (f = 1)</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>bookings (f = 2)</td>
<td>15</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>bookings (f = 3)</td>
<td>94</td>
<td>60</td>
<td>36</td>
</tr>
<tr>
<td>bookings (f = 4)</td>
<td>0</td>
<td>0</td>
<td>50</td>
</tr>
<tr>
<td>Revenue</td>
<td>50.600</td>
<td>24.000</td>
<td>24.400</td>
</tr>
</tbody>
</table>

- Differences in dependent and independent demand structure
- Revenue of NL\_1 is evaluated at the MNL situation (same fare classes open)
- Revenue of NL\_2 is obtained from letting the model decide on open fare classes

\(^1\) evaluated at MNL
## Low Demand Load Factor

<table>
<thead>
<tr>
<th></th>
<th>MNL</th>
<th>NL1²</th>
<th>NL2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacity</td>
<td>150</td>
<td>150</td>
<td>150</td>
</tr>
<tr>
<td>Customers</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>bookings $f = 1$</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>bookings $f = 2$</td>
<td>6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>bookings $f = 3$</td>
<td>34</td>
<td>21</td>
<td>25</td>
</tr>
<tr>
<td>bookings $f = 4$</td>
<td>13</td>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>Revenue</td>
<td>22,000</td>
<td>10,200</td>
<td>10,600</td>
</tr>
</tbody>
</table>

²evaluated at MNL
Summary

- Advantages from discrete choice models
  - Price elasticities
  - Coefficient values may be adjusted over time
  - Modeling stochastically dependent demand

- Mathematical model for simulation of individual utility values
  - Allows application of more complex discrete choice models other than MNL
  - Reveals differences in revenues obtained
Thank you very much for your attention!