Choice Based Airline Revenue Management Modeling with Flexible Substitution Patterns

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Introduction
Research Contribution

Development of a revenue maximization model

- that decides on the optimal seat allocation
- that sets the optimal prices for each offered seat
- based on integrating demand from discrete choice models

Assumptions

- Static model
- Single resource with capacity $C$
- Multiple products $\rightarrow$ fare classes
- Two compartments $\rightarrow$ business and economy class
- Demand is modeled as fare class choice $\rightarrow$ utility maximization
- Stochastically dependent demand
Motivation

- Simplifying assumptions in static single-resource RM models
  - Demand for different fare classes are stochastically independent
  - Demand does not depend on other available fare classes
- Constant substitution patterns between fare classes

Realization

1. Simulation based model for revenue optimization
2. Demand from a DCM\textsuperscript{a} with not constant substitution patterns

\textsuperscript{a}Discrete Choice Model
Demand Model
Discrete Choice Models

- Allow to model multiproduct RM with *flexible*\(^1\) capacity
- Stochastic demand models
- Information assymetrie between seller and buyer

**Seller**

- Only partial information on choice decision
- Observes individual utility values from actual or stated choice decisions

**Buyer**

- Full information on choice decision
- Choice decision is influenced by
  - (i) Characteristics of the alternatives and the customer
  - (ii) Customers evaluation of alternatives
- Stochastically dependent demand structures

\(^1\)Ability to offer different products using the initial capacity \(C\) [BC03].
Demand model

Properties

Multinomial Logit Model (MNL)

\[ U_f = V_f + \varepsilon_f \quad \forall f \]

- Constant substitution
- \( \varepsilon_f \) is iid EV distributed
- Identical cross elasticities
- IIA property
- [TVR04]

Nested Logit Model (NL)

\[ U_f = V_f + \left( \varepsilon_m + \varepsilon_{fm} \right) \quad \forall f \in C_m|m \]

- No constant substitution
- Correlation of pairs of \( f \)
- Generalization of MNL
- More realistic choice behaviour
- [WK01]
Application I: Modelling Demand - Fare Class Choice
Utility Function

Considered function of deterministic utility

\[ U_f = \beta_{f,asc} + \beta_{price} \cdot x_{f,price} + \beta_{f,prp} \cdot x_{prp} + \beta_{f,gen} \cdot x_{gender} + \varepsilon_f \quad \forall f \]

- Generation of synthetic datasets based on \( U_f \)
- NL and MNL models differ according to \( \varepsilon_f \)
- Coefficient values are provided \( \rightarrow \) "true" values

Results

- Two populations with 10,000 individuals each
- Choice behaviour according to NL and MNL models
- Datasets exhibit expected substitution patterns
- Assumption: NL population exhibits "true" behaviour
Choice set contains 4 fare class alternatives $i$:

- Regular (1) and discount (2) fare in Business Class
- Regular (3) and discount (4) fare in Economy Class
- No choice/Outside Alternative (OA) (5)

Provided coefficient values:

- $\beta_{price} = -0.0040$
- $\beta_{1,prp} = 2.0$, $\beta_{2,prp} = 1.5$, $\beta_{3,prp} = 1.0$, $\beta_{4,prp} = 0.5$, $\beta_{5,prp} = 0$
- $\beta_{1,gen} = 0.8$, $\beta_{2,gen} = 0.5$, $\beta_{3,gen} = 0.2$, $\beta_{4,gen} = -0.1$, $\beta_{5,gen} = 0$

Assignment of alternatives to nests:

- Nest 1 = {1,2}
- Nest 2 = {3,4}
- Nest 3 = {5}
Figure: Tree structure for fare class choice in the MNL (left) and NL (right)
1. Draw random sample of 2000 observations from **NL** population
2. Use sample as dataset for estimation
3. Estimate NL model from NL sample → NL coefficients
4. Estimate MNL model from NL sample → MNL coefficients
5. Put NL coefficients in optimization model with NL utility function
6. Put MNL coefficients in optimization model with MNL utility function
7. Compare results
## Estimation Results

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>NL estimate</th>
<th>t-stat</th>
<th>MNL estimate</th>
<th>t-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\beta_{asc,1}$</td>
<td>0.321</td>
<td>0.69</td>
<td>0.229</td>
<td>0.37</td>
</tr>
<tr>
<td>2</td>
<td>$\beta_{asc,2}$</td>
<td>1.83</td>
<td>4.40</td>
<td>2.51</td>
<td>8.23</td>
</tr>
<tr>
<td>3</td>
<td>$\beta_{asc,3}$</td>
<td>1.32</td>
<td>6.83</td>
<td>1.16</td>
<td>5.81</td>
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<tr>
<td>4</td>
<td>$\beta_{asc,4}$</td>
<td>1.98</td>
<td>14.31</td>
<td>2.15</td>
<td>17.44</td>
</tr>
<tr>
<td>5</td>
<td>$\beta_{price}$</td>
<td>-0.00459</td>
<td>-8.64</td>
<td>-0.00559</td>
<td>-15.75</td>
</tr>
<tr>
<td>6</td>
<td>$\beta_{prp,1}$</td>
<td>1.08</td>
<td>3.47</td>
<td>1.30</td>
<td>3.21</td>
</tr>
<tr>
<td>7</td>
<td>$\beta_{prp,2}$</td>
<td>0.361</td>
<td>1.95</td>
<td>0.347</td>
<td>1.80</td>
</tr>
<tr>
<td>8</td>
<td>$\beta_{prp,3}$</td>
<td>0.145</td>
<td>0.97</td>
<td>0.231</td>
<td>1.42</td>
</tr>
<tr>
<td>9</td>
<td>$\beta_{prp,4}$</td>
<td>-0.193</td>
<td>-1.60</td>
<td>-0.212</td>
<td>-1.74</td>
</tr>
<tr>
<td>10</td>
<td>$\beta_{gen,1}$</td>
<td>2.40</td>
<td>6.35</td>
<td>2.68</td>
<td>5.34</td>
</tr>
<tr>
<td>11</td>
<td>$\beta_{gen,2}$</td>
<td>1.47</td>
<td>7.53</td>
<td>1.45</td>
<td>7.16</td>
</tr>
<tr>
<td>12</td>
<td>$\beta_{gen,3}$</td>
<td>1.07</td>
<td>6.64</td>
<td>1.22</td>
<td>7.31</td>
</tr>
<tr>
<td>13</td>
<td>$\beta_{gen,4}$</td>
<td>0.516</td>
<td>4.14</td>
<td>0.477</td>
<td>3.80</td>
</tr>
<tr>
<td>14</td>
<td>NestA</td>
<td>1.73</td>
<td>1.77(^1)</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>15</td>
<td>NestB</td>
<td>1.36</td>
<td>1.71(^1)</td>
<td>–</td>
<td>–</td>
</tr>
</tbody>
</table>

\(^1\) t-test against 1
Application II: Simulation based RM Optimization Model
Preface

Generate utility values for two populations of individuals

- Choice behavior based on MNL and NL models
- Maximum utility determines choice decision
- Linear model formulation
- General formulation in terms of applied DCM
- Calculation of optimal seat allocation per fare class
  - Decision on fare classes offered to an individual
  - Fare classes are offered in a revenue maximizing manner
  - Choice sets vary across individuals
- Calculation of optimal price per fare class \( f \)
- Solved with Cplex in GAMS version 23.9
Definitions

Sets

- \( S \): Simulated sets of individuals
- \( F \): Set of fare classes indexed \( f \), no choice/OA \( f = 5 \)
- \( N \): Set of simulated individuals indexed \( n \ \forall s \in S \)

Parameters

- \( \bar{p}_f \): Upper bound for price variable in fare class \( f \)
- \( L \): Sufficiently large number
- \( A \): Sufficiently large number to guarantee \( u_{snf} > 0 \)
- \( C \): Seat capacity of considered resource
- \( d_{snf} \): Deterministic utility plus \( A \), without \( \beta_{price} \cdot p_f \)
- \( \beta_{price} \): Price coefficient
Variables

\( u_{snf} \) Utility individual \( n \) receives from choosing fare class \( f \) in \( s \)

\( \bar{u}_{sn} \) Maximum utility value for individual \( n \) in \( s \)

\( p_{f} \) Price for a ticket in fare class \( f \) in \( s \)

\( \pi_{snf} \) Price individual \( n \) pays for a ticket in fare class \( f \) in \( s \)

\( b_{f} \) Maximum amount of bookings in fare class \( f \) in \( s \)

\( y_{snf} \) \( = 1 \), if individual \( n \) chooses fare class \( f \) in \( s \) (0, else)

\( x_{snf} \) \( = 1 \), if \( f \) is offered to \( n \) in \( s \) (0, else)

\( Z \) Objective value
Revenue Optimization

Objective Function

\[
\max Z = \frac{1}{|S|} \cdot \sum_{s} \sum_{n} \sum_{f} \pi_{snf}
\]  

(1)

Price & Utility

\[
\begin{align*}
    u_{snf} - d_{snf} - \beta_{price} \cdot p_{f} & \leq 0 & \forall s, n, f \\
    u_{snf} - d_{snf} - \beta_{price} \cdot p_{f} + L \cdot (1 - x_{snf}) & \geq 0 & \forall s, n, f \\
    \pi_{snf} - \bar{p}_{f} \cdot y_{snf} & \leq 0 & \forall s, n, f \\
    \pi_{snf} - p_{f} & \leq 0 & \forall s, n, f \\
    \tilde{u}_{sn} - u_{snf} & \geq 0 & \forall s, n, f \\
    u_{snf} - L \cdot x_{snf} & \leq 0 & \forall s, n, f \\
    \tilde{u}_{sn} - u_{snf} - L \cdot (1 - y_{snf}) & \leq 0 & \forall s, n, f
\end{align*}
\]  

(2) (3) (4) (5) (6) (7) (8)
Remaining Constraints

\[ \sum_{n} y_{snf} - b_f \leq 0 \quad \forall s, f < 5 \quad (9) \]

\[ \sum_{f} y_{snf} = 1 \quad \forall s, n \quad (10) \]

\[ y_{snf} - x_{snf} \leq 0 \quad \forall s, n, f \quad (11) \]

\[ x_{snf} - x_{sn-1f} \leq 0 \quad \forall s, n > 1, f < 5 \quad (12) \]

\[ b_f \geq 0 \quad \forall f \quad (13) \]

\[ p_f \geq 0 \quad \forall f \quad (14) \]

\[ \pi_{snf} \geq 0 \quad \forall s, n, f \quad (15) \]

\[ x_{snf} \in \{0, 1\} \quad \forall s, n, f \quad (16) \]

\[ y_{snf} \in \{0, 1\} \quad \forall s, n, f \quad (17) \]
Results
Overview

- Results for varying price coefficient $\beta_p$
- $\beta_p$ changes according to start value (MNL/NL) by $+0.0005$
  $\Rightarrow$ Change in individuals price sensitivity
  - Demand is constant with 10 generated individuals per simulation run
  - Available Capacity is $C = 5$
  - Simulated sets of individuals $S = 20$
  - Comparison for NL and MNL models
Open fare classes for number of individuals

Figure: Open fare classes for MNL (left) and NL (right) utilities
Results

Chosen fare classes on average

Figure: Number of bookings for MNL (left) and NL (right) utilities
Prices payed on average

Figure: Prices payed on average for MNL (left) and NL (right) utilities
Results

Average Revenues

<table>
<thead>
<tr>
<th>MNL Revenue</th>
<th>NL Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.00559</td>
<td>-0.00539</td>
</tr>
<tr>
<td>-0.00519</td>
<td>-0.00499</td>
</tr>
<tr>
<td>-0.00479</td>
<td>-0.00459</td>
</tr>
<tr>
<td>-0.00439</td>
<td>-0.00419</td>
</tr>
<tr>
<td>-0.00399</td>
<td>-0.00379</td>
</tr>
<tr>
<td>-0.00359</td>
<td>-0.00339</td>
</tr>
</tbody>
</table>
Direct Elasticities

Figure: Direct elasticities for MNL (left) and NL (right) utilities
Cross Elasticities

Figure: Cross elasticities for MNL (left) and NL (right) utilities
Thank you very much for your attention!
### Coefficients

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>MNL</th>
<th>NL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>true value</td>
<td>estimate</td>
</tr>
<tr>
<td>$\beta_{asc,1}$</td>
<td>0.50</td>
<td>0.869</td>
</tr>
<tr>
<td>$\beta_{asc,2}$</td>
<td>1.50</td>
<td>1.590</td>
</tr>
<tr>
<td>$\beta_{asc,3}$</td>
<td>1.50</td>
<td>1.510</td>
</tr>
<tr>
<td>$\beta_{asc,4}$</td>
<td>2.00</td>
<td>1.970</td>
</tr>
<tr>
<td>$\beta_{asc,5}$</td>
<td>0.00</td>
<td>.</td>
</tr>
<tr>
<td>$\beta_{price}$</td>
<td>-0.0040</td>
<td>-0.0042</td>
</tr>
<tr>
<td>$\beta_{purpose,1}$</td>
<td>2.00</td>
<td>1.960</td>
</tr>
<tr>
<td>$\beta_{purpose,2}$</td>
<td>1.50</td>
<td>1.640</td>
</tr>
<tr>
<td>$\beta_{purpose,3}$</td>
<td>1.00</td>
<td>1.130</td>
</tr>
<tr>
<td>$\beta_{purpose,4}$</td>
<td>0.50</td>
<td>0.600</td>
</tr>
<tr>
<td>$\beta_{purpose,5}$</td>
<td>0.00</td>
<td>.</td>
</tr>
<tr>
<td>$\beta_{gender,1}$</td>
<td>0.80</td>
<td>0.726</td>
</tr>
<tr>
<td>$\beta_{gender,2}$</td>
<td>0.50</td>
<td>0.494</td>
</tr>
<tr>
<td>$\beta_{gender,3}$</td>
<td>0.20</td>
<td>0.226</td>
</tr>
<tr>
<td>$\beta_{gender,4}$</td>
<td>-0.10</td>
<td>-0.0517</td>
</tr>
<tr>
<td>$\beta_{gender,5}$</td>
<td>0.00</td>
<td>.</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>1.9</td>
<td>.</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>2.10</td>
<td>.</td>
</tr>
<tr>
<td>$\mu_3$</td>
<td>1.00</td>
<td>.</td>
</tr>
</tbody>
</table>

**Table:** True coefficient values for the dataset generation of MNL and NL
### Substitution Patterns I

**MNL**

<table>
<thead>
<tr>
<th>Fare class</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market share</td>
<td>5.10</td>
<td>10.50</td>
<td>45.30</td>
<td>31.00</td>
<td>8.10</td>
</tr>
</tbody>
</table>

- Market shares when all fare classes are available

<table>
<thead>
<tr>
<th>Fare class</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market share</td>
<td>6.96</td>
<td>14.97</td>
<td>65.56</td>
<td>0.00</td>
<td>12.51</td>
</tr>
</tbody>
</table>

- Alternative 4 is no longer available
- Market shares of 1, 2, 3 and 5 rise proportionally
- Ratio of substitution between any pair of alternatives is constant

→ **Example:** ratio of alternatives $\frac{1}{3} = \frac{5.10}{45.30} = \frac{6.96}{65.56} = 0.11$
## Substitution Patterns II

### Fare class and Market share

<table>
<thead>
<tr>
<th>Fare class</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market share</td>
<td>2.93</td>
<td>6.67</td>
<td>40.41</td>
<td>28.29</td>
<td>21.70</td>
</tr>
</tbody>
</table>

- Nest 1 = \{1,2\}, Nest 2 =\{3,4\}, Nest 3 =\{5\}

<table>
<thead>
<tr>
<th>Fare class</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market share</td>
<td>3.53</td>
<td>8.26</td>
<td>60.03</td>
<td>0.00</td>
<td>28.18</td>
</tr>
</tbody>
</table>

- Market share of alternative in the same nest (=3) rises proportionally
- Flexible substitution *across* nests, constant substitution *within* nests

→ **Example:** ratio of alternatives \(\frac{1}{3} = \frac{2.93}{40.41} = 0.073 \neq \frac{3.53}{60.03} = 0.059\)

→ **Example:** ratio of alternatives \(\frac{1}{2} = \frac{2.93}{6.67} = \frac{3.53}{8.26} = 0.43\)
References

