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Revenue Sharing for Vessel Pooling

Part Three of Research Project

“Analysis of Containership Pooling and Benefit Sharing Scheme”

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EXECUTIVE SUMMARY

This report is *Part Three* of the project "*Analysis of Containership Pooling and Benefit Sharing Scheme*", The purpose of this part is to propose a new revenue-sharing scheme for the ship pools in the containership charter market, and to compare it to the traditional Pool Points approach. The traditional Pool Points method allocates the total pool revenue proportional to the contributed capacity of each owner. In addition to some operational difficulties, another essential drawback of this method is that it neglects the Economies of Scale effect from different owners. The new method is proposed based on Cooperative Game concepts, which offer attracting characteristics such as stability and fairness, and also incorporate the real impact from each owner joining the coalition. In this study, Shapley value is combined with the Core method.

For example, if we assume the increase of market share has constant (positive) marginal impact on price, i.e. constant returns to scale (CRS), the Shapley value method gives the same results as the Pool Point method, and every owner in the pool obtains the revenue share proportionally to its size.

However, it is more likely that the market share size has decreasing marginal positive impact on the charter price, i.e. decreasing returns to scale (DRS). In another word, the owner's size has a positive effect on charter price, but this effect will get weaker with size increase. In this situation, the Shapley value method will allocate to the larger ship owner a revenue share which is higher than the share given by the Pool Point method. The SV method is considered fairer since the larger owner contributes higher revenue increase than the smaller owner. It is also a stable solution which means the smaller owner still has incentive to join the alliance, since he also gets a higher revenue than the situation if he does not join.

1. INTRODUCTION

This report is *Part Three* of the project “*Analysis of Containership Pooling and Benefit Sharing Scheme*”, The purpose of this part is to propose a new revenue-sharing scheme for the ship pools in the containership charter market. The background of this study has been discussed in detail in *Part One*, including the status of global container shipping market and the basic concepts, benefits, and concerns of ship pooling among tramp owners in the charter market. In *Part Two* of the report, an empirical analysis on containership charter price, especially the impact of market share size, has been conducted.

As *Part Three* of the report, we propose a new revenue sharing scheme and compare it to the traditional Pool Points approach. The traditional Pool Points method allocates the total pool revenue proportional to the contributed capacity of each owner. In addition to some operational difficulties, another essential drawback of this method is that it neglects the additional benefits or costs added by certain owners because of the Economies of Scales effect (EOS) or Diseconomies of Scale effect (DEOS).

The new method is proposed based on Cooperative Game concepts, which offer attracting characteristics such as stability and fairness, and also incorporate the Economies of Scale effect. In this study, Shapley value is used.

If the owner’s size has a positive effect on charter price, and this effect will get weaker with size increase, the Shapley value method should allocate a revenue share to the larger ship owner which is higher in proportional to its capacity share.

2. TRADITIONAL POOL POINT METHOD

The traditional way of sharing revenue in the ship pool is to assign a pool point (PP) to each vessel based on its basic characteristics. The pool points will then determine each ship’s share of the net earnings of all ships in the pool.

There are totally m ship owners in the market; and owner O_i has n_i number of vessels. So totally there are $\sum_{i=1}^{m} n_i$ vessels in the market. Vessel V_{ij} belongs to the ith owner in the pool and is assigned with a Pool Point $\boldsymbol{w_{ij}}$ based on its characteristics. Vessel V_{ij} obtains a price of $\boldsymbol{p_{ij}}$ from the charter market. The revenue share of this vessel from the pool is $\boldsymbol{R_{ij}}$:

$$\boldsymbol{R_{ij}} = \frac{w_{ij}}{\sum_{i,j} w_{ij}} \left(\sum_{i,j} p_{ij}\right), \text{ where } \boldsymbol{i = 1, 2, \ldots m;\ j = 1, 2, \ldots n_i}$$

Equation 1: PP Method

In practice, one difficulty of this method is that the criteria for awarding pool points is not always set out in a transparent way and some pool agreements leave a good deal of discretion to the committee of the participants. Most usually the pool agreement states that the relevant committee will allocate or adjust points based essentially on performance characteristics (like TEU capacity, design speed, etc.) of the vessel and the time during which the ship has been in the pool.

But additionally, another essential drawback of this traditional approach is that it neglects the additional benefits or costs added by certain owners because of the Economies of Scales effect (EOS) or Diseconomies of Scale effect (DEOS). Revenue sharing schemes which possess characteristics such as stability and fairness will be discussed based on Cooperative Game Theory and be tested with empirical numbers in comparison with existing methods.

3. COOPERATIVE GAME THEORY: SHAPLEY VALUE AND CORE

3.1. Shapley value

A commonly used method in cooperative game is called Shapley value (SV). Some of the properties of Shapley value make it a very attracting method: There always exists one and only one unique Shapley value solution to a game; and the solution is considered as "fair" because the Shapley value for a player is essentially the weighted average of the contributions the player makes to all possible coalitions, while the weight depends on the number of players *n* and the number of members in each coalition.

The formula to calculate the Shapley value payoff $\emptyset_i$ to the *owner* i is given as below.

$\boldsymbol{\emptyset_i = \sum_K \big(v(K) - v(K\backslash\{i\})\big) \frac{(k-1)!(n-k)!}{n!}}$, where v is the **characteristics function** of the game, and $v(K)$ gives the expected value of coalition K, which is represented by the total market payoff of all the vessels in the coalition.

Equation 2: Shapley Value

The formula could be interpreted as follows. Totally *n* owners are assumed to enter the game in a random order. When owner i arrives, he gets the extra amount value (revenue increase or cost saving or both) he brings to the game, which is *v(K) – v(K\{i})* assuming there are *K-1* players ahead of him. The probability that player *i* enters the game after any *K-1* players and before any other *n-K* players could be calculated as $(k-1)!\,(n-k)!/n!$.

Since by itself, Shapley value does not guarantee a stable solution, it has to be checked whether the Shapley value is part of the Core which are the sets of stable solutions. We introduce the concept of Core in the next subsection.

3.2. The Core

In cooperative game, the Core of a game *v*, is the set of solution vectors that are not dominated for any other solutions; otherwise it is not stable.

If a solution vector is in the Core, there does not exist a subcoalition of players that could make all of its members at least as well off and one member strictly better off. If a feasible allocation *x* is not in the Core, there is a subcoalition *S* such that the players in *S* could all do strictly better than in *x* by cooperating together and dividing the worth *v(S)* among themselves.

Plainly speaking, if we find a solution that is in the Core, this solution assigns payoffs to all the members of the coalition such that no one can further increase its payoff by forming other types of subcoalitions.

A payoff vector x is in the Core if and only if $\sum_{i=1}^{n} x_i = v(N)$ and $\sum_{i \in S}^{n} x_i \geq v(K)$ for all $K \in N$.

The first condition (Efficiency Condition) requires that total repay to all the players in the game should equal to the grand coalition's value and is a Pareto optimality condition. The second condition requires that the payoff solution is not dominated by any other solutions.

4. POOL REVENUE FUNCTION

The charter price function format reflects the market reality of market share size's impact on prices. Two kinds of formats are tested in this study.

4.1. Linear price function

The charter price function is a linear function of the market share of the owner/broker size, holding all other factors fixed, assuming the increase of market share has constant marginal impact on price:

$p_i = a * \left(\frac{n_i}{N}\right)$, n_i is the number of vessels owned by owner O_i, N is the total number of vessels in the charter market, assuming all vessels are of the same size.

When the market size is N, the price is a which is the maximum possible charter price. Without influencing the results and for convenience of presentation, a is set as 1.

Under this setting, two conclusions are made (See approval in Appendix A):

1. The Shapley value method gives the same results as the Pool Point method.
2. Every owner in the pool obtains the revenue share proportionally to its size.

4.2. Exponential price function

If the market share size has decreasing marginal positive impact on the charter price, the price function is an exponential function, with the power $\alpha \in (0, 1)$:

$$p_i = a * \left(\frac{n_i}{N}\right)^{\alpha}.$$

Same as above, a is set as 1. When α is positive, the characteristics function is supperadditive, and the derived Shapley value is in the core. If the characteristics function v is superadditive, the Shapley value must be individually rational, in the sense that $\emptyset_i(v) \geq v(\{i\}), \forall i \in N$.

Under this setting, market share increase has a decreasing marginal impact on charter price. In another word, although larger owners obtain higher prices, this effect gets weaker as the market share size increases towards 100% of the market. Analysis and findings are presented below (See approval in Appendix B)

1. The traditional PP method still allocates the total pool revenue proportional to the capacity of each owner.
2. By the SV method, ship owners who contribute a relatively large amount of vessel capacity into the pool will get revenue share higher than its capacity share (i.e. the PP method).

In conclusion, for $\alpha \in (0,1)$, if owner $O_1$is larger than owner O_2 (i.e. $n_1 > n_2$), the Shapley value method will allocate a revenue share to owner O_1 which is higher than its share by the Pool Point method. And the results are reversed when alpha is larger than 1 or when owner 1 is the smaller owner.

The SV method is considered fairer since the larger owner contributes higher revenue increase than the smaller owner. It is also a stable solution because it is part of the Core solution set. That means the smaller owner still has incentive to join the alliance, since he also gets a higher revenue than the situation if he does not join ($\emptyset_2 > \frac{{n_2}^2}{N}$).

APPENDIX A: LINEAR PRICE FUNCTION

Assume in the market there are two ship owners. Owner O_1 has n_1 vessels, and Owner O_2 has n_2 vessels. O_1 is larger than O_1, i.e. $n_1 > n_2$. The vessels are assumed to very similar and will be given the same pool point $\boldsymbol{w}$ by the traditional method. (This will not affect the general conclusion of the analysis.) The two owners form a coalition with $N = n_1 + n_2$ vessels.

Pool Point (PP) method:

- Unit revenue share per vessel for Owner O_1 based on Pool Points is (with $a = 1$):
  $UPP_1 = \frac{\boldsymbol{w_i}}{\sum_{i,j} \boldsymbol{w_{ij}}} \left(\sum_{i,j} \boldsymbol{p_{ij}}\right) = \frac{w}{N*w} N = 1.$
- Similarly, revenue share based on Pool Point for owner 2 is:
  $UPP_2 = 1.$

Shapley value (SV) method:

With two owners, there are 3 possible "alliances": $\{O_1\}$, $\{O_2\}$, $\{O_1, O_2\}$. Applying the SV equation, the values of these alliances are obtained:

$V(\{O_i\}) = \frac{{n_i}^2}{N}$, where $i = 1, 2, 3.$

$V(\{O_1, O_2\}) = \frac{N^2}{N} = N.$

Revenue share based on Shapley value method for Owner O_1 is calculated as:

$$\emptyset_1 = (V(\{O_1\}) - 0)\frac{(1-1)!(3-1)!}{3!} + \left(V(\{O_1, O_2\}) - V(\{O_2\})\right)\frac{(2-1)!(3-2)!}{3!} = n_1.$$

The unit revenue share per vessel for Owner O_1 based on SV:

$USV_1 = \frac{1}{n_1}\emptyset_1 = 1$. Similarly, owner 2 has the same unit revenue share per vessel.

APPENDIX B: EXPONENTIAL PRICE FUNCTION

Pool Point (PP) method gives the same allocation as the linear price function.

Shapley Value (SV) method

The possible “alliances”: $\{O_1\}$, $\{O_2\}$, $\{O_1, O_2\}$. Applying **Error! Reference source not found.**the SV equation, the values of these alliances are obtained:

$$V(\{O_i\}) = \frac{{n_i}^{\alpha+1}}{N^\alpha}, \text{ where } i = 1, 2, 3.$$

$$V(\{O_1, O_2\}) = \frac{N^{\alpha+1}}{N^\alpha} = N.$$

$$\emptyset_1 = (V(\{O_1\}) - 0)\frac{(1-1)!(2-1)!}{2!} + \big(V(\{O_1, O_2\}) - V(\{O_2\})\big)\frac{(2-1)!(2-2)!}{2!} = \frac{(n_1+n_2)^{\alpha+1} + {n_1}^{\alpha+1} - {n_2}^{\alpha+1}}{2N^\alpha}.$$

Comparison of SV method with PP method

Using the SV method, the larger owner gets $\emptyset_1$, while using PP method, it gets n_1. Next, compare the two allocation results: $\emptyset_1 - n_1 = \frac{(n_1+n_2)^\alpha(n_2-n_1)+{n_1}^{\alpha+1}-{n_2}^{\alpha+1}}{2(n_1+n_2)^\alpha} = \frac{1}{2}\{n_2(1-q^\alpha) - n_1[1-(1-q)^\alpha]\} = \frac{n_1+n_2}{2}\{q(1-q^\alpha) - (1-q)[1-(1-q)^\alpha]\}$, where $q = \frac{n_2}{n_1+n_2}$.

Set $f(q) = q(1-q^\alpha) - (1-q)[1-(1-q)^\alpha]$. We get $f''(q) = \alpha(1+\alpha)[(1-q)^{\alpha-1} - q^{\alpha-1}]$. Because $n_2 < n_1$, $q < \frac{1}{2}$, $\alpha \in \emptyset_1 - n_1(0,1)$, and $f(0) = f(1) = 0$. Therefore, $f''(q) < 0$, and $f(q)$ is always positive when $n_2 < n_1$.

In conclusion, for $\alpha \in (0,1)$, if owner $O_1$is larger than owner O_2 (i.e. $n_1 > n_2$), the Shapley value method will allocate a revenue share to owner O_1 which is higher than its share by the Pool Point method (i.e. $\emptyset_1 - n_1 > 0$).

Visual illustrations

To visually illustrate the comparison of these two methods, the difference $\emptyset_1 - n_1$ is revised as $\emptyset_1 - n_1 = \frac{(1+x)^\alpha n^\alpha(1-x)n + n^{\alpha+1} - x^{\alpha+1}n^{\alpha+1}}{2(1+x)^\alpha n^\alpha} = \frac{(1+x)^\alpha(1-x)+x^{\alpha+1}-1}{2(1+x)^\alpha} * n = g(x) * n$, where $n_2 = n, n_1 = x * n, x > 1$. With given $\alpha \in (0,1)$, $g(x)$ can be shown in Figure B1 below (created in MATLAB).

Figure B1 also shows that the results are reversed when alpha is larger than 1 or when owner 1 is the smaller owner.

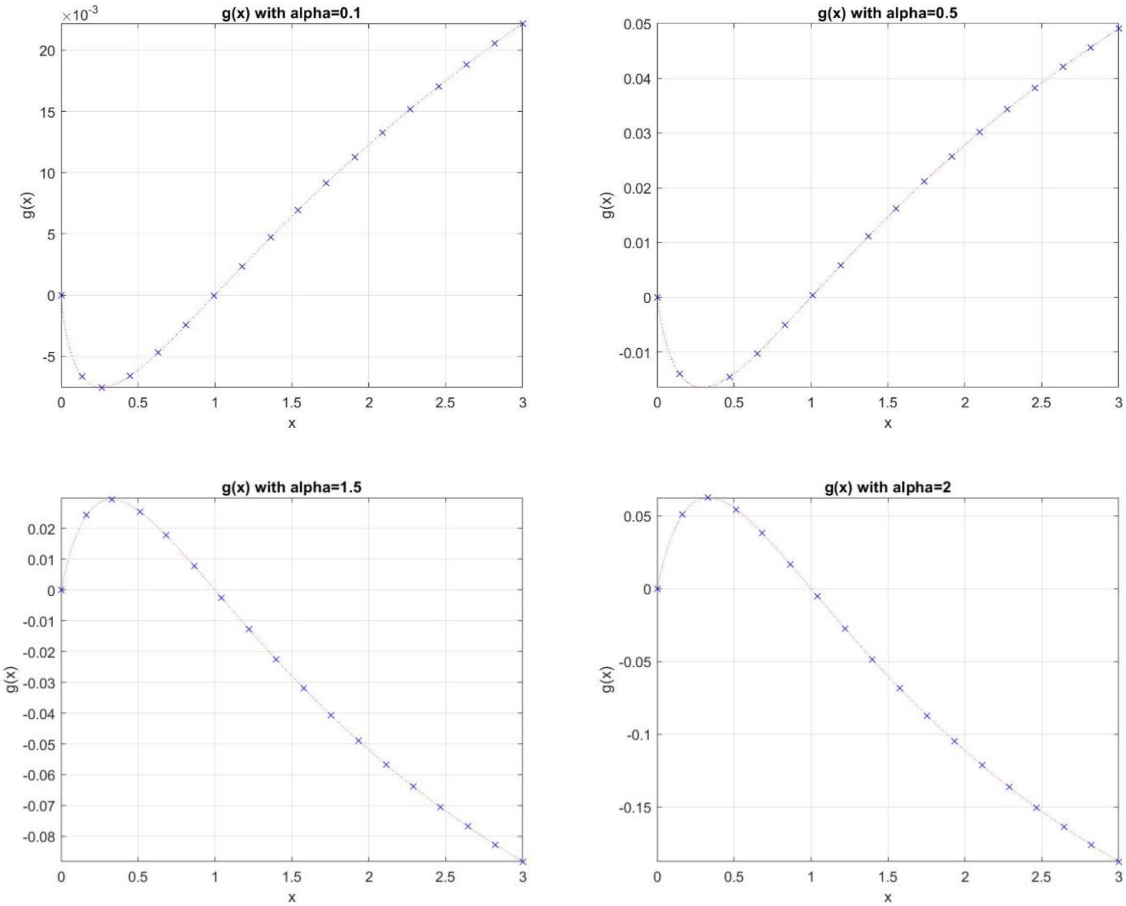

Figure B1: Two method comparison