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An improved Groff criterion for myopic lot sizing in both regular *and* sporadic demand

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Preface

In its core, this working paper is a translation of the German version Stadtler (2022). Apart from some minor editorial changes the following modifications and additions have been made:

- The term *modified Groff* is substituted by *Groff-zero*.
- A new section reporting some results of a computational test documented in Dujesiefken, Stadtler, and Voigt (2024) is included. Here, we present an excerpt which compares the solution quality of the Silver/Meal, the (original) Groff, the (new) Groff-zero heuristic, and a variant of the Wagner/Whitin algorithm.
- Further references have been added.
- An appendix contains a pseudo code of the Groff-zero heuristic.

Abstract

Myopic lot-sizing heuristics are still used in operational practice today due to their simplicity and comprehensibility. In addition, the resulting setup and inventory holding costs are often only insignificantly higher than those of an exact solution (e.g. using the Wagner/Whitin algorithm), especially in the case of rolling schedules. In numerical tests, the Silver/Meal heuristic and the Groff heuristic in particular have shown a very high solution quality. However, this only applies to regular demand. For sporadic demand, a modification of the Silver/Meal heuristic was presented in Silver/Miltenburg (1984). Such a modification is not known for the Groff heuristic; this is to be made up for in the present working paper.

1. Introduction and literature

In the following, we consider the dynamic, single-item lot sizing problem (LSP) without capacity restrictions. It is based on deterministic net period demands d_t for a planning interval divided into t=1..T periods. The setup costs and the inventory holding costs in the planning interval are to be minimized while the net period demands are met. Myopic lot sizing heuristics to solve the LSP are based on the assumption that the fixed setup cost rate *sc* is period-independent and that the inventory at the end of a period is valued at the inventory holding cost rate *hc*. With this information, the following mathematical model can be formulated:

Model

$$\min \sum_{t=1}^{T} \left(sc \cdot f(X_t) + hc \cdot I_t \right)$$

$$s.t.$$

$$I_{t-1} + X_t = d_t + I_t \quad \forall\, t$$

$$f(X_t) = 1, if \;\; X_t > 0; 0\, otherwise \;\; \forall\, t$$

$$X_t \geq 0 \quad \forall t$$

$$I_t \geq 0 \quad \forall t$$

Variables

I_t end-of-period inventory, period t (note: I_0 = 0)

X_t lot size in period t

The objective function minimizes the setup and holding costs in the planning interval. The first constraint type represents the inventory balance constraints. Function $f(X_t)$ returns a value of “1” if there is a lot size in period t (“0” otherwise). Finally, the non-negativity constraints are included.

To solve the above model, Wagner and Whitin developed an exact solution algorithm based on dynamic programming as early as 1958 (Wagner and Whitin 1958). Its computational complexity is $O(T^2)$. Later, researchers – like Wagelmanns, van Hoesel, and Koolen (1992) – presented improved algorithms that even solve the above model in linear time $O(T)$. Nevertheless, myopic lot sizing heuristics are still widely used today due to their simplicity and comprehensibility. Myopic lot sizing heuristics are characterized by the fact that they progressively combine period demands into one lot from one period to the next until a predefined criterion takes effect. This forward procedure also leads to linear complexity $O(T)$. Well-known representatives of myopic lot sizing heuristics are the least-unit cost rule (unknown source), the part-period rule (DeMatteis 1968), the Silver/Meal (Silver/Meal 1973), and the Groff heuristic (Groff 1978).

Lot sizing heuristics to solve the LSP have been subjected to extensive numerical tests by Zoller/Robrade (1987). Test data with different regular demand functions have been used for this purpose. Regular demand exists when all period demands are positive, while sporadic demand is characterized by a high share of periods with no demand (subsequently called zero-demand periods). The Silver/Meal and Groff heuristics stood out from their study due to their excellent solution quality.

Only a few contributions are available for sporadic demand. These include Silver/Miltenburg (1984), Silver/Peterson (1985, footnote on p. 234), Knolmayer (1986) and Yilmaz (1992). The focus of Silver/Miltenburg (1984) is on improving the solution quality of the Silver/Meal heuristic through two improvement steps. In addition, a simple extension of the Silver/Meal heuristic for sporadic demand is presented right at the beginning of the paper. Knolmayer examines several myopic heuristics and modifies them such that "... all heuristics were adjusted in such a way that each lot arrives in a period with positive demand" (Knolmayer 1986). Knolmayer uses numerical tests to show that this modification generally leads to an improved solution quality of the myopic lot sizing heuristics under consideration. Yilmaz (1992) propagates the incremental order policy. However, this heuristic is not considered further here, as it has already shown relatively poor results for regular demand (see Zoller/Robrade 1987).

To the best of our knowledge, a targeted modification of the Groff criterion (also) for sporadic demand is lacking in the literature. This working paper aims to close this gap. The aim is to modify the Groff criterion in such a way that it generates identical solutions to the (original) Groff heuristic in the case of regular demand and also provides cost-effective solutions in the case of sporadic demand.

Section 2 derives the (new) Groff-zero criterion in case of sporadic demand. Furthermore, a numerical example is included to demonstrate the application of the Groff-zero criterion. Also, we compare the (original) Groff criterion with the Groff-zero criterion and show that both are identical in case of regular demand. In Section 3 we show some results from a computational study conducted by Dujesiefken, Stadtler, and Voigt (2024), in which several lot sizing methods including Groff-zero have been tested in rolling schedules. Finally, we have a summary and an outlook (Section 4). An appendix contains a pseudo code of the Groff-zero heuristic with linear complexity $O(T)$.

2. The Groff-zero heuristic

2.1 Derivation of the Groff-zero criterion

To better understand the derivation of the Groff-zero criterion, we have illustrated the symbols used to identify certain periods on the time axis in Figure 1.

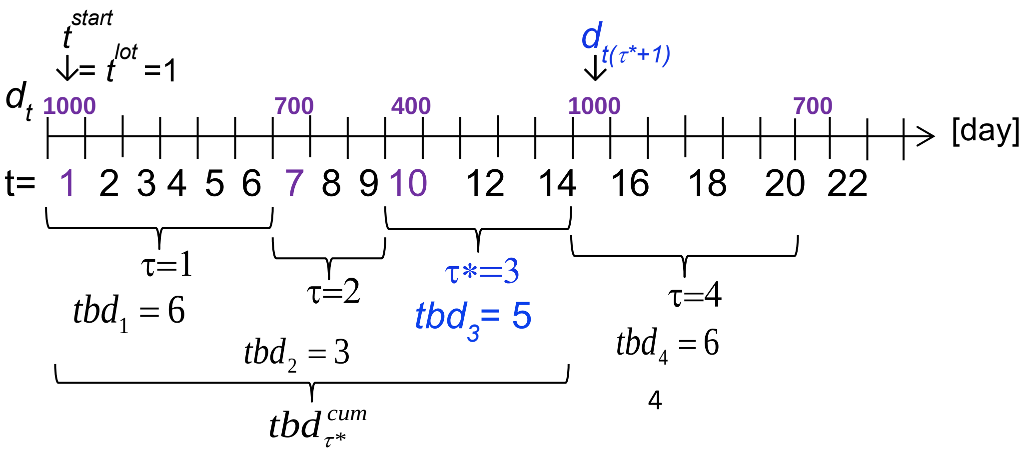

Figure 1: Example of a time series with sporadic period demand

A demand cycle τ always begins with a positive demand and ends in the previous period of the next positive demand, its duration is the time between demands tbd_{τ}. A positive demand can be followed by one or more zero-demand periods. Demand cycles form the basis to calculate the range of a lot that covers the demand cycles $1..\tau$:

$$tbd_{\tau}^{cum} = \sum_{i=1}^{\tau} tbd_{i} \tag{6}$$

We start the derivation of the Groff-zero criterion by looking at the difference in setup costs per period with a lot of range τ and a range of τ + 1 demand cycles:

$$\Delta SC_{\tau,\tau+1} = \frac{sc}{tbd_{\tau}^{cum}} - \frac{sc}{tbd_{\tau+1}^{cum}} \tag{7}$$

After merging and shortening the two fractions, the following ratio results:

$$\frac{tbd_{\tau+1} \cdot sc}{tbd_{\tau}^{cum} \cdot tbd_{\tau+1}^{cum}} \tag{8}$$

Next, we consider the difference in inventory holding costs per period when the range of a lot is extended by the next demand cycle τ + 1.

$$\Delta H_{\tau,\tau+1} = \frac{\sum_{i=1}^{\tau+1} tbd_{i-1}^{cum} \cdot hc \cdot d_{t+tbd_{i-1}^{cum}}}{tbd_{\tau+1}^{cum}} - \frac{\sum_{i=1}^{\tau} tbd_{i-1}^{cum} \cdot hc \cdot d_{t+tbd_{i-1}^{cum}}}{tbd_{\tau}^{cum}} \tag{9}$$

The Groff criterion applies as soon as the increase in inventory holding costs per period is greater than the reduction in setup costs per period for the first time due to an increase in the lot size´s range of coverage.

$$\Delta H_{\tau,\tau+1} > \Delta SC_{\tau,\tau+1} \tag{10}$$

As a result, the lot size in period *t* covering $\tau *$ demand cycles is:

$$x_{t} = \sum_{i=1}^{\tau *} d_{t+tbd_{i-1}^{cum}} \tag{11}$$

Further transformations lead to the following ratio:

$$\frac{tbd_{\tau}^{cum} \cdot H_{t+\tau+1} - tbd_{\tau+1}^{cum} \cdot H_{t+\tau}}{tbd_{\tau}^{cum} \cdot tbd_{\tau+1}^{cum}} \tag{12}$$

$H_{t+\tau}$ $(H_{t+\tau+1})$ represents the inventory holding costs for a lot size covering τ (τ + 1) demand cycles:

$$H_{t+\tau} = \sum_{i=1}^{\tau} tbd_{i-1}^{cum} \cdot hc \cdot d_{t+tbd_{i-1}^{cum}} \tag{13}$$

$$H_{t+\tau+1} = H_{t+\tau} + tbd_{\tau}^{cum} \cdot hc \cdot d_{t+tbd_{\tau}^{cum}} \tag{14}$$

The (exact) Groff-zero criterion (also) for sporadic demand then results as follows:

$$tbd_{\tau}^{cum} \cdot H_{t+\tau+1} - tbd_{\tau+1}^{cum} \cdot H_{t+\tau} > tbd_{\tau+1} \cdot sc \tag{15}$$

In the case of equidistant demand cycles *tbd* and a constant demand *d* in periods with positive demand, the above equation can be simplified, resulting in the following equation for the increase in inventory costs:

$$\Delta H_{\tau,\tau+1} = \frac{\tau \cdot (\tau+1) \cdot hc \cdot d}{2 \cdot (\tau+1) \cdot \tau} \tag{16}$$

It should be noted here that the derivation of the (original) Groff criterion also assumes a constant period demand (see e.g. Baciarello et al. 2013). The equation for calculating the increase in setup costs can be adjusted as follows:

$$\Delta SC_{\tau,\tau+1} = \frac{sc}{tbd \cdot (\tau+1) \cdot \tau}$$

(17)

The combination of the two equations leads to the Groff-zero criterion (also) for sporadic demand, which is very similar to the (original) Groff criterion.:

$$\frac{1}{2} \cdot tbd \cdot hc \cdot d > \frac{sc}{\tau^* \cdot (\tau^*+1)} \tag{18}$$

In a final step, the above formula is transferred to the dynamic demand so that the modified, approximated Groff criterion (also for the sporadic demand) is obtained:

$$\frac{1}{2} \cdot tbd_{\tau^*} \cdot hc \cdot d_{t(\tau^*+1)} > \frac{sc}{\tau^* \cdot (\tau^*+1)} \tag{19}$$

with

τ^*	number of demand cycles covered by the current lot
$t(\tau^*+1)$	indicates the period of positive demand of demand cycle τ^*+1 (which is not included in the current lot having a range of τ^* cycles)
tbd_{τ^*}	duration of the last demand cycle covered by the current lot

2.2 A numericel example

The application of the Groff heuristic will be illustrated using a numerical example. We start with the Groff-zero heuristic (Table 1) followed by the (original) Groff heuristic (Table 2). The inventory holding cost rate *hc* = 0.01 and the setup cost rate *sc* = 100 are chosen. The (sporadic) period demands within the planning interval *T* = 15 and the solutions achieved can be observed in the two tables below.

Lot no.	t	tstart	τ	tbd_τ	tbd_τ^{cum}	$d_t > 0$	additional inv. costs	marginal setup costs	criterion		lot size	inv. costs	setup costs
							$0.5 \cdot tbd_\tau \cdot hc \cdot d_{t(\tau+1)}$	$sc/(\tau \cdot (\tau+1))$	?		q		
1. Los	1	1	1	6	6	1000	21,0000	50,0000	$<$	false	2100	0	100
	7		2	3	9	700	6,0000	16,6667	$<$	false		42	
	10		3	5	0	400	25,0000	8,3333	$>$	true		36	
2. Los	15	15	1			1000	0,0	50,0000	$<$	false	1000		100
											hc^{sum} =	78	200 = sc^{sum}
											total =	278	

Table 1: Calculation of lot sizes by the Groff-zero heuristic

Lot no.	t	t^{start}	τ^{p}	d_t	*add.* inv. costs	marginal setup costs	criterion	lot size	inv.costs	setup costs
					$0.5 \cdot hc \cdot d_{\tau^{p}+1}$	$sc / (\tau^{p} \cdot (\tau^{p}+1))$	?	q		
1. Los	1	1	1	1000	0,000	50,0000	< false	1000	0	100
	2		2	0	0,0000	16,6667	"			
	3		3	0	0,0000	8,3333	"			
	4		4	0	0,0000	5,0000	"			
	5		5	0	0,0000	3,3333	"			
	6		6	0	3,5000	2,3810	> true			
2. Los	7	7	1	700	2,0000	50,0000	< false	1100		100
	8		2	0	0,0000	16,6667	"			
	9		3	0	2,0000	8,3333	"			
	10		4	400	0,0000	5,0000	"		12	
	11		5	0	0,0000	3,3333	"			
	12		6	0	0,0000	2,3810	"			
	13		7	0	0,0000	1,7857	"			
	14		8	0	5,0000	1,3889	< true			
3. Los	15	15	1	1000	0,0000	50,0000	< false	1000	0	100
								hc^{sum} =	12	300 = sc^{sum}
								total =	312	

Tabelle 2: Calculation of lot sizes by the (original) Groff heuristic

A comparison of the solutions of the two heuristics shows that the (original) Groff heuristic proposes three lots in the planning interval with setup costs of 300 [MU] and relatively low inventory holding costs of 12 [MU]. The Groff-zero heuristic, on the other hand, suggests two lots and leads to setup and inventory holding costs of 278 [MU]. The (original) Groff heuristic therefore causes additional costs of 12.2%.

2.3 Comparison of the (original) Groff criterion and the Groff-zero criterion

If we now compare the (original) Groff criterion

$$\frac{1}{2} \cdot hc \cdot d_{tstart+\tau^{p^*}} > \frac{sc}{\tau^{p^*} \cdot (\tau^{p^*} + 1)} \tag{20}$$

with the (new) Groff-zero criterion (inequality (19)), we observe that both are identical for regular demand. The (original) Groff criterion is based on the range of a lot measured in number of periods τ^{p^*} and the duration of a (last) period, which is set to "1". In contrast, the Groff-zero criterion is based on the number of covered demand cycles τ^* and the duration of the last demand cycle tbd_{τ^*} within the range of the lot (which may be different from "1"). In the case of regular demand, the number of demand cycles corresponds to the number of periods within the range of a lot (assuming no zero-demand period). The criteria for terminating a lot size therefore do not differ for regular demand. In the case of sporadic demand, however, the number of demand cycles is smaller than the number of periods within the range of a lot. The Groff-zero criterion "ignores" zero-demand periods and therefore usually leads to lots with longer ranges.

In order to achieve a high solution quality for both regular and sporadic demand, only two changes need to be made to the (original) Groff heuristic. Firstly, the demand cycles are considered simply by "ignoring" zero-demand periods in the calculations (see Appendix). Secondly, the (original) Groff criterion (20) must be replaced by the Groff-zero criterion (19) above. The Groff-zero heuristic therefore has a linear complexity $O(T)$, just like the (original) Groff heuristic.

3. A comparative performance evaluation of the Groff-zero heuristic

Subsequently we present an excerpt from the paper of Dujesiefken, Stadtler, and Voigt (2024) where a number of algorithms that solve the LSP have been compared in a large computational test. Their computational test adopts a number of ideas and data developed by Zoller and Robrade (1987). They distinguish between three different time series of deterministic demand: constant, systematic, and erratic. Other parameters adopted are the part periods (resulting in an expected range of a lot of 1 to 6 periods) and the demand pattern (e.g. the interval of fluctuations for erratic demand).

In order to test the performance of the lot sizing algorithms in the context of zero-demand periods Dujesiefken, Stadtler, and Voigt (2024) disaggregate these demand series from an assumed (regular) weekly demand to daily demand with seven days a week and five working days. Three different shares of zero-demand periods in a week are considered (equivalent to two, three, or four working days with zero demand and no demand on weekends). Within each week the working days with positive demand are selected randomly.

In order to limit the number of test instances resulting from the combination of parameter values, Dujesiefken, Stadtler, and Voigt (2024) only apply a single planning horizon, namely T=13 weeks, which is transformed into T=7*13= 91 days for the case of sporadic demand.

As in Zoller and Robrade (1987), the algorithms are tested in rolling schedules with a flexible evaluation horizon to avoid the planning horizon effect. Hence, the evaluation horizon ranges between 40 and 50 periods for the weekly schedules and 280 to 350 days for sporadic demand. In total there are 102 test instances for regular demand and 306 test instances for sporadic demand. Each test instance is run 10 times. Table 3 shows the number of different parameter values considered.

Table 3 The number of parameter values and resulting test instances

	Regular (weeks)			Sporadic (days)		
	Constant	Systematic	Erratic	Constant	Systematic	Erratic
Planning horizon	1	1	1	1	1	1
Part periods	6	6	6	6	6	6
Demand pattern	1	11	5	1	11	5
Share of zero-demand periods	-	-	-	3	3	3
No. test instances	6	66	30	18	198	90

We will limit our presentation of test results to a single performance indicator, the relative additional costs of a lot sizing algorithm compared to the corresponding minimal total costs possible. This performance indicator has to be calculated for each run of a test instance (for more details see Zoller Robrade (1987, p. 227). Furthermore, we focus on the results of the Silver/Meal heuristic (as presented in Silver/Miltenburg (1984)), the (original) Groff and the Groff-zero heuristic as well as the Wagner/Whitin algorithm with the look-beyond-the-planning-horizon extension (Stadtler 2000) (abbreviated WW-lb).

First, we address regular demand. As expected, mean relative additional costs of the Silver/Meal, Groff and Groff-zero heuristic are almost the same (assuming that a difference of 0.1% is negligible) and are largest in the case of *regular* erratic demand (1.1%). WW-lb outperforms the myopic heuristics slightly with mean relative additional costs of 0.8% in the case of regular erratic demand.

Second, test results for sporadic demand are presented in Table 4. We observe that the Groff-zero heuristic has much lower mean relative additional costs than its original counterpart. Actually, mean relative additional costs can be reduced remarkably from 15.7% to 5.0%.

Table 4 Mean relative additional costs in [%] for different shares of zero-demand periods

Share of zero-demand per.	Silver/Meal	Groff	Groff-zero	WW-lb
57%	9.3143	8.998	4.843	0.092
71%	8.329	14.108	4.440	0.133
86%	1.562	23.845	5.611	0.146
Average	6.402	15.650	4.964	0.124

When looking at the performance of the Silver/Meal and the Groff-zero heuristic it is striking that mean relative additional costs of the Silver/Meal heuristic decrease the larger the share of zero-demand periods becomes while there is no obvious trend for the Groff-zero heuristic. The best performance, however, has been achieved by WW-lb, irrespective of the share of zero-demand periods. Its mean relative additional costs compared to the minimum are negligible (0.1%).

Still, the use of the Silver/Meal or Groff-zero heuristic in operational practice might be justified despite the considerably better performance of the WW-lb algorithm, due to the errors that are usually made when forecasting demand or calculating the “correct” setup and holding cost rates.

4. Summary and outlook

We have derived the (new) Groff-zero criterion that is designed for both regular *and* sporadic demand. The basic idea here is to consider demand cycles instead of individual periods. A demand cycle always begins with a positive demand and ends in the previous period of the next positive demand.

We have been able to show that the (original) Groff criterion can be interpreted as a special case of the Groff-zero criterion, provided that there is no demand cycle with a zero-demand period. Consequently, the Groff-zero criterion results in the same solution quality as the (original) Groff criterion in the case of regular demand.

Computational tests with rolling schedules reported in Dujesiefken, Stadtler, and Voigt (2024) reveal that the Groff-zero heuristic clearly outperforms the (original) Groff heuristic in case of zero-demand periods. Even more the Groff-zero heuristic results in lower total setup and holdings costs than the Silver/Meal heuristic provided the share of zero-demand periods does not exceed 71%. Consequently, we recommend pretests with past demand series if one of these two heuristics is to be used in operational practice.

Future research is under way considering "nearly" sporadic demand patterns, i.e. those in which there are periods of relatively high demand followed by periods of relatively low demand. These demand patterns arise in the case of cyclical deliveries to bulk buyers with simultaneous daily fulfillment of relatively small spare parts requirements.

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Appendix: A pseudo code for the Groff-zero heuristic

$t^{lot} := T + 1$

$sc^{sum} := 0.$

$inv^{sum} := 0.$

$for\ t = 1..T\ do$

$\quad if\ d_t > 0\ then$

$\qquad t^{start} := t$

$\qquad t^{lot} := t$

$\qquad t^{tbd} := t$

$\qquad \tau := 1$

$\qquad X_t := d_t$

$\qquad sc^{sum} := sc$

$\qquad break$

$\quad end - if$

$end - do$

$for\ t = t^{start}..T-1\ do$

$\quad if\ d_{t+1} > 0\ then$

$\qquad tbd_\tau := (t+1) - t^{tbd}$

$\qquad if\ tbd_\tau \cdot d_{t+1} \cdot \mathrm{hc} > \dfrac{2 \cdot \mathrm{sc}}{\tau \cdot (\tau + 1)}\ \ \mathrm{then}$

$\qquad\quad t^{lot} := t + 1$

$\qquad\quad t^{tbd} := t + 1$

$\qquad\quad \tau := 1$

$\qquad\quad X_{t+1} := d_{t+1}$

$\qquad\quad sc^{sum} := sc^{sum} + sc$

$\qquad else$

$\qquad\quad inv^{sum} := inv^{sum} + (t + 1 - t^{lot}) \cdot d_{t+1}$

$\qquad\quad t^{tbd} := t + 1$

$\qquad\quad \tau := \tau + 1$

$\qquad\quad X_{t^{lot}} := X_{t^{lot}} + d_{t+1}$

$\qquad end - if$

$\quad end - if$

$end - do$

$total := hc \cdot inv^{sum} + sc^{sum}$

Note

Numerical operations can be reduced drastically by calculating arrays before starting the heuristic, e.g. $\rho(\tau) := 2 \cdot sc / (hc \cdot (\tau \cdot (\tau + 1)))\ \ \forall \tau = 1..T$ to be used in the Groff-zero criterion or $t1(t) := t + 1\ \ \forall t = 1..T-1$ both valid for any subsequent rolling schedule.

Explanations

- Rows 1 to 14 initialize constants and determine the first period with positive net demand. This will be the start of the first lot size.
- Rows 15 to 31 depict the core of the Groff-zero heuristic.
- Only periods with a positive net demand are considered for starting a (new) lot size.
- Row 17 calculates the duration of demand cycle τ (time between demands)
- Row 18 contains the Groff-zero criterion.
- If the Groff-zero criterion applies, a new lot starts in period t+1 (rows 19 to 23).
- If not, the lot size will be increased by the net demand in period t+1 (rows 25 to 28).
- Finally, total setup and holding costs in the planning interval are calculated (row 32).