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Risk Margin for the Runoff of Non-Life Insurance Reserves

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Valuation and cash flow prediction

Insurance contracts generate (random) insurance payment cash flows.

Aim:

1. **Predict** and **value** these insurance payment cash flows!

These predictions and valuations should always be based on the latest information available.

Reserves and provisions

- Prediction of the outstanding liabilities gives the **(claims) reserves** or the **(claims) provisions**.

- These reserves (or provisions)

  - should suffice to meet **all future payments**
    $\Longrightarrow$ **reserves and solvency**;

  - are the basis for future **premium calculations**;

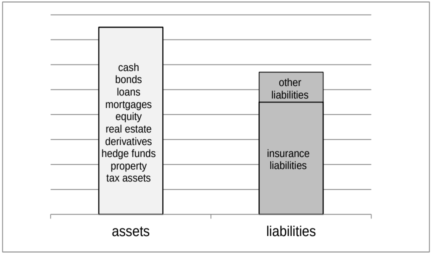
  - determine the **risk management process**.

- These reserves are the most important insurance position at all.

What are the main requirements that these reserves should fulfill?

Full balance sheet approach

- Balance sheet should be valued in a **market-consistent** way:
  - market values where available;
  - marked-to-model approach otherwise.



- **Insurance liabilities:**
  No market values.

- Therefore for reserves:
  Market-consistent prediction of outstanding insurance liabilities in a marked-to-model approach.

- What does this exactly mean?

Technical provisions

- Solvency II Directive 2009/138/EC:
  Insurance liabilities should be valued at the amount for which they could be exchanged between knowledgeable willing parties in an arm's length transaction.

- The resulting amount is called **technical provisions**.

- The technical provisions are the sum of the **best-estimate reserves** and the **risk margin**.

- What are best-estimate reserves? Why a risk margin?

$$\boxed{\textbf{deterministic best-estimate reserves} \iff \textbf{stochastic claims payments}}$$

Best-estimate reserves for outstanding liabilities

- "The best-estimate should correspond to the probability weighted average of future cash flows taking account of time value of money."

Mathematical model: $(\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{t\in\mathbb{N}})$ filtered probability space:

- $\mathcal{F}_t$ information available at time $t \in \mathbb{N}$;
- $\boldsymbol{\varphi} = (\varphi_t)_{t\in\mathbb{N}}$ stochastic discount function (financial deflator);
- $\mathbf{X} = (X_t)_{t\in\mathbb{N}}$ insurance liability cash flow, $(\mathcal{F}_t)_{t\in\mathbb{N}}$-adapted.

Best-estimate reserves at time $k \in \mathbb{N}$ for liabilities $(X_t)_{t>k}$ (see [3])

$$\mathcal{R}_k(\mathbf{X}) = \sum_{t>k} \mathbb{E}\left[\frac{\varphi_t}{\varphi_k} X_t \middle| \mathcal{F}_k \right].$$

Stochastic discounting of best-estimate reserves

- Find appropriate stochastic model $(\Omega, \mathcal{F}, \mathbb{P}, (\mathcal{F}_t)_{t \in \mathbb{N}})$ and
  - $\boldsymbol{\varphi} = (\varphi_t)_{t \in \mathbb{N}}$ stochastic discount function (financial deflator),
  - $\mathbf{X} = (X_t)_{t \in \mathbb{N}}$ insurance liability cash flow,

  and calculate best-estimate reserves at time $k \in \mathbb{N}$

$$\mathcal{R}_k(\mathbf{X}) = \sum_{t>k} \mathbb{E}\left[\left. \frac{\varphi_t}{\varphi_k} X_t \right| \mathcal{F}_k \right].$$

- In general, with $(r_t^{(k)})_{t \geq 0}$ risk-free term structure at time k,

$$\mathcal{R}_k(\mathbf{X}) \neq \sum_{t>k} \frac{1}{\left(1 + r_{t-k}^{(k)}\right)^{t-k}} \mathbb{E}\left[X_t \middle| \mathcal{F}_k \right],$$

  due to options, guarantees, inflation, etc.

Risk margin or market-value margin MVM

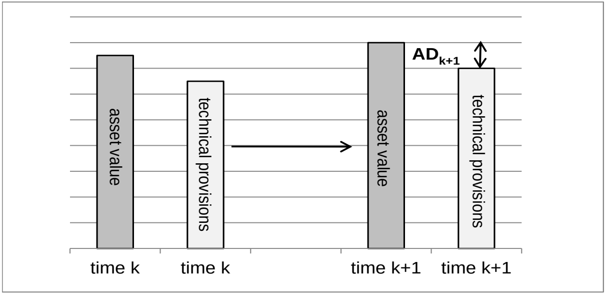
deterministic best-estimate reserves $\Longleftrightarrow$ stochastic claims payments

- How reliable is the prediction $\mathcal{R}_k(\mathbf{X})$ for $(X_t)_{t>k}$?
- A risk averse agent asks for a **risk margin** (**market-value margin**) for possible shortfalls in this prediction.
- The **technical provisions** (market-consistent value) for the outstanding insurance liabilities are then given by

$$\mathcal{R}_k^{(*)}(\mathbf{X}) = \mathcal{R}_k(\mathbf{X}) + \mathrm{MVM}_k(\mathbf{X}).$$

- **How should we calculate this risk margin $\mathrm{MVM}_k(\mathbf{X})$?**

Solvency at time k



Solvency is given at time k iff:

1. asset values cover technical provisions $\mathcal{R}_k^{(*)}(\mathbf{X})$ at time k, and
2. the possibility of an asset deficit $\mathrm{AD}_{k+1} > 0$ at time $k+1$ is sufficiently small (measured by an appropriate risk measure).

Conclusions for solvency calculation

We need a stochastic model that allows for:

1. calculation of best-estimate reserves $\mathcal{R}_k(\mathbf{X})$ at time k;
2. calculation of risk margin $\mathrm{MVM}_k(\mathbf{X})$ at time k;
3. modeling of asset deficit AD_{k+1} at time $k+1$.

Note: Everything holds true for life and non-life insurance.

Risk margin in non-life insurance

- We give 3 different approaches for the calculation of the risk margin.
- In non-life insurance one often **assumes** that claims payments $\mathbf{X}$ are **independent** from financial market developments. This implies

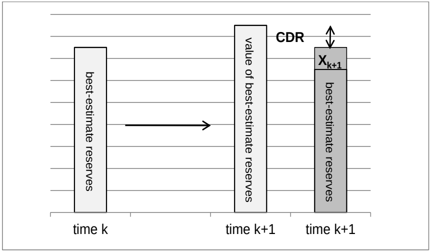
$$\mathcal{R}_k(\mathbf{X}) \;=\; \sum_{t>k} \mathbb{E}\left[X_t \middle| \mathcal{F}_k\right] \; P(k,t),$$

  with $P(t,k)$ price of the zero coupon bond with maturity t at time k.

- The **claims development result (CDR)** at time $k+1$ is given by

$$\mathrm{CDR}(k+1) = \left(\sum_{t>k} \mathbb{E}\left[X_t \middle| \mathcal{F}_k\right] P(k+1,t)\right) - \left(X_{k+1} + \mathcal{R}_{k+1}(\mathbf{X})\right).$$

Claims development result



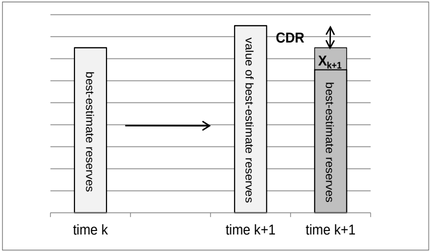
The CDR at time $k+1$

$$\mathrm{CDR}(k+1) = \left(\sum_{t>k} \mathbb{E}\left[X_t \middle| \mathcal{F}_k \right] P(k+1,t) \right) - \left(X_{k+1} + \mathcal{R}_{k+1}(\mathbf{X}) \right)$$

considers the **update of information** $\mathcal{F}_k \mapsto \mathcal{F}_{k+1}$:

- $\mathrm{CDR}(k+1) < 0$: additional capital is needed;
- $\mathrm{CDR}(k+1) > 0$: we have a gain in the P&L statement.

Risk margin: approach 1



Cost-of-capital risk margin:

1. Calculate the solvency capital requirement (SCR) for possible shortfalls in this CDR position.

   $\Longrightarrow$ This provides **risk measure** ρ_k for accounting year $k+1$.

2. The risk margin should be related to this SCR ρ_k.

Market-value margin 1 (current solvency practice)

The first cost-of-capital (CoC) approach defines the risk margin as

$$\mathrm{MVM}_k^{(1)}(\mathbf{X}) \;=\; r_{\mathrm{CoC}} \cdot \sum_{t>k} w_t \cdot \rho_k,$$

where

- ρ_k risk measure (SCR) for possible shortfalls in $\mathrm{CDR}(k+1)$;
- r_{CoC} cost-of-capital rate $> r_0^{(k)}$ (risk-free rate at time k);
- $(w_{k+1}, w_{k+2}, w_{k+3}, \ldots)$ expected runoff of the outstanding liabilities $(X_{k+1}, X_{k+2}, X_{k+3}, \ldots)$ at time k.

Interpretation. The risk margin from this CoC approach should reflect the reward for risk bearing, i.e. an investor provides the SCRs $w_t \cdot \rho_k$ and therefore receives a rate of return $r_{\mathrm{CoC}} > r_0^{(k)}$ on these SCRs.

Difficulties with the market-value margin 1

$$\mathrm{MVM}_k^{(1)}(\mathbf{X}) \;=\; r_{\mathrm{CoC}} \cdot \sum_{t>k} w_t \cdot \rho_k,$$

1. Choice of risk measure ρ_k (SCR):
   - runoff or going-concern view?
   - stand-alone or diversified?
   - per line-of-business or whole insurance portfolio?
2. Choice of r_{CoC}? Is $r_{\mathrm{CoC}} = r_0^{(k)} + 6\%$ appropriate?
3. $w_t \cdot \rho_k$ is **not** a risk-based approximation to the SCRs ρ_t in accounting years $t > k$.

Risk margin: approach 2 (Salzmann-W. [1])

For simplicity, we choose nominal reserves, i.e. $P(k,t) \equiv 1$.

Then

$$\mathrm{CDR}(k+1) = \sum_{t>k} \mathbb{E}\left[X_t \middle| \mathcal{F}_k \right] - \sum_{t>k} \mathbb{E}\left[X_t \middle| \mathcal{F}_{k+1} \right].$$

This implies for $t > k$

$$\mathbb{E}\left[\mathrm{CDR}(t) \middle| \mathcal{F}_k \right] = 0,$$

and, moreover,

$\mathrm{CDR}(k+1),\ \mathrm{CDR}(k+2), \ldots$ are **uncorrelated** (**not independent**).

This follows because successive best-estimate predictions are martingales.

Split of total uncertainty

Uncorrelatedness provides the **total prediction variance decomposition**

$$\mathrm{Var}\left(\sum_{t>k} \mathrm{CDR}(t) \middle| \mathcal{F}_k \right) = \sum_{t>k} \mathrm{Var}\left(\mathrm{CDR}(t) \middle| \mathcal{F}_k \right). \tag{1}$$

- Formula (1) gives a **risk-based allocation** of the total uncertainty measured by the prediction variance to individual accounting years.
- In many models we can explicitly calculate $\mathrm{Var}\left(\mathrm{CDR}(t) \middle| \mathcal{F}_k \right)$, e.g. Γ-Γ Bayes chain ladder model of Salzmann-W. [1].

Market-value margin 2 (split of total uncertainty)

The second cost-of-capital (CoC) approach defines the risk margin as

$$\mathrm{MVM}_k^{(2)}(\mathbf{X}) = r_{\mathrm{CoC}} \cdot \sum_{t>k} \Phi \cdot \mathrm{Var}\left(\mathrm{CDR}(t) \middle| \mathcal{F}_k\right)^{1/2},$$

where

- $\rho_t = \Phi \cdot \mathrm{Var}\left(\mathrm{CDR}(t) \middle| \mathcal{F}_k\right)^{1/2}$ standard deviation risk measure for possible shortfalls in $\mathrm{CDR}(t)$ on security level $\Phi > 0$;
- r_{CoC} cost-of-capital rate $> r_0^{(k)}$ (risk-free rate at time k).

Remarks.

- This provides a risk-adjusted market-value margin.
- Other (multi-period) risk measures are too involved and do not lead to applicable solutions (nested simulations).

Risk margin: approach 3 (economic approach)

Idea: The technical provisions $\mathcal{R}_k^{(*)}(\mathbf{X})$ should be a market-consistent price for the outstanding insurance liabilities:

- a rational investor calculates under risk aversion a margin for non-hedgeable (insurance technical) risks.
- In economic theory this is usually done with **utility functions** and/or with **probability distortions**.

For non-life claims reserving **probability distortions** are a feasible way, see W. et al. [3], Section 2.6.

Market-value margin 3 (chain-ladder framework 1/2)

For a particular chain-ladder claims reserving model (W.-Merz [4]):

- **Best-estimate reserves:**

$$\mathcal{R}_k(\mathbf{X}) \; = \; C_k \; \left(\prod_{t>k} f_t - 1 \right),$$

where C_k are the cumulative payments at time k and f_t are the so-called chain-ladder factors.

- **Technical provisions:**

$$\mathcal{R}_k^{(*)}(\mathbf{X}) \; = \; C_k \; \left(\prod_{t>k} f_t^{(*)} - 1 \right),$$

where $f_t^{(*)}$ are the **risk-adjusted chain-ladder factors**.

Market-value margin 3 (chain-ladder framework 2/2)

- **Risk margin:**

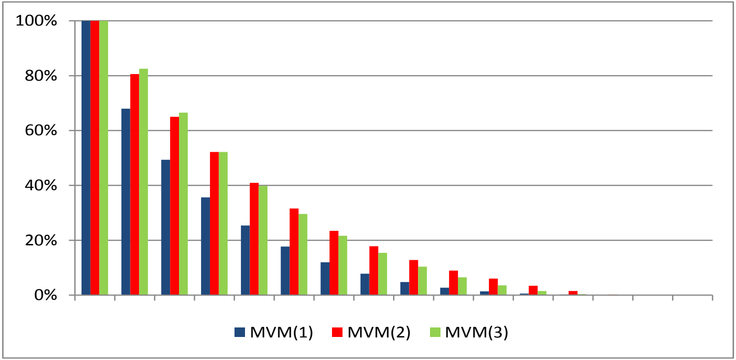
$$\mathrm{MVM}_k^{(3)}(\mathbf{X}) \;=\; \mathcal{R}_k^{(*)}(\mathbf{X}) - \mathcal{R}_k(\mathbf{X}) \;=\; C_k \;\left(\prod_{t>k} f_t^{(*)} - \prod_{t>k} f_t\right).$$

- **Risk-adjusted chain-ladder factors:** a sensible choice is

$$f_t^{(*)} \;=\; (f_t - 1)\;\exp\left\{h_t(\alpha)\right\} + 1 \;>\; f_t,$$

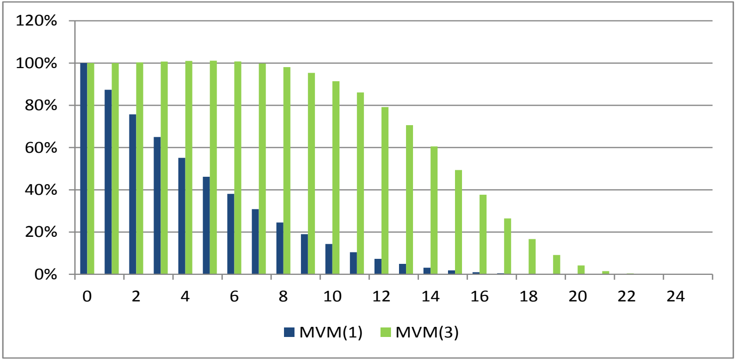
for $h_t(\alpha) > 0$ a positive function of the risk aversion parameter α.

Case studies: private liability insurance



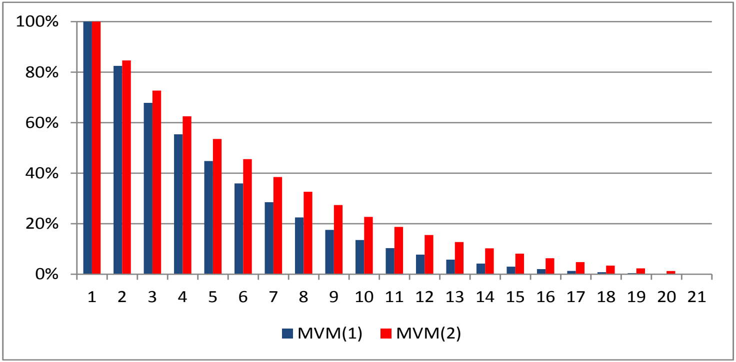
Expected runoff pattern of risk margins $\mathrm{MVM}_k^{(i)}(\mathbf{X})$ for approaches $i = 1, 2, 3$.

Case studies: life-time annuity



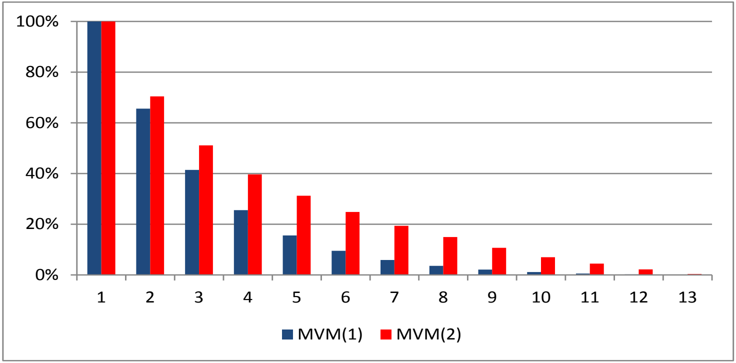
Expected runoff pattern of risk margins $\mathrm{MVM}_k^{(i)}(\mathbf{X})$ for approaches $i = 1, 3$.

Case studies: motor third party liability insurance



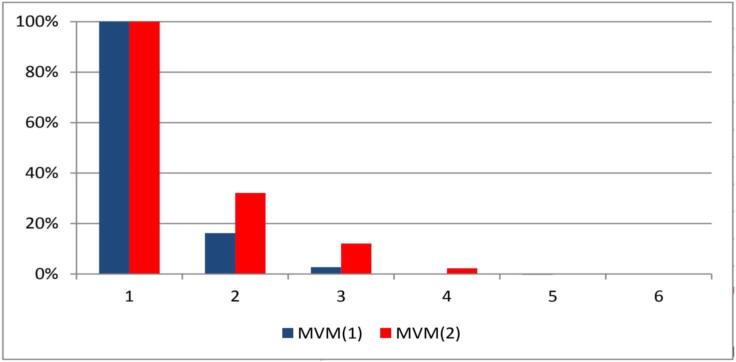
Expected runoff pattern of risk margins $\mathrm{MVM}_k^{(i)}(\mathbf{X})$ for approaches $i = 1, 2$.

Case studies: general liability insurance



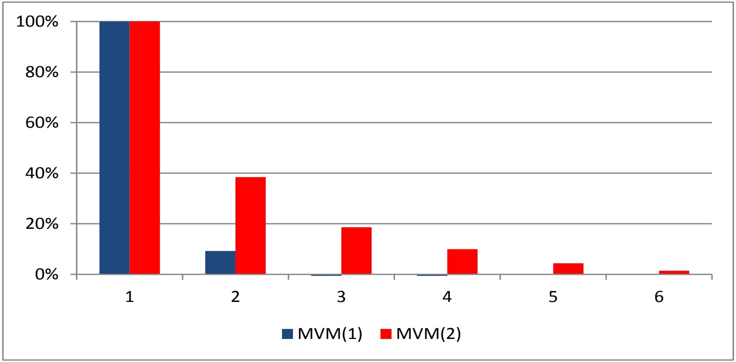
Expected runoff pattern of risk margins $\mathrm{MVM}_k^{(i)}(\mathbf{X})$ for approaches $i = 1, 2$.

Case studies: property insurance



Expected runoff pattern of risk margins $\mathrm{MVM}_k^{(i)}(\mathbf{X})$ for approaches $i = 1, 2$.

Case studies: health insurance



Expected runoff pattern of risk margins $\mathrm{MVM}_k^{(i)}(\mathbf{X})$ for approaches $i = 1, 2$.

References

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