Choice Based Airline Revenue Management Modeling with Flexible Substitution Patterns

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Content

Introduction

Research Contribution

Development of a revenue maximization model

- that decides on the optimal seat allocation
- that sets the optimal prices for each offered seat
- based on integrating demand from discrete choice models

Assumptions

- Static model
- Single resource with capacity C
- Multiple products $\rightarrow$ fare classes
- Two compartments $\rightarrow$ business and economy class
- Demand is modeled as fare class choice $\rightarrow$ utility maximization
- Stochastically dependent demand

Approach

Motivation

- Simplifying assumptions in static single-resource RM models
  - → Demand for different fare classes are stochastically independent
  - → Demand does not depend on other available fare classes
- Constant substitution patterns between fare classes

Realization

1. **Simulation** based model for revenue optimization
2. Demand from a DCM[a] with **not constant substitution patterns**

[a] Discrete Choice Model

Demand Model

Discrete Choice Models

- Allow to model multiproduct RM with *flexible*[1] capacity
- Stochastic demand models
- **Information assymetrie** between seller and buyer

Seller

- Only partial information on choice decision
- Observes individual utility values from actual or stated choice decisions

Buyer

- Full information on choice decision
- Choice decision is influenced by
  - (i) Characteristics of the alternatives and the customer
  - (ii) Customers evaluation of alternatives
- Stochastically dependent demand structures

[1] Ability to offer different products using the initial capacity C [BC03].

Properties

Multinomial Logit Model (MNL)

$$U_f = V_f + \varepsilon_f \quad \forall\, f$$

- Constant substitution
- ε_f is iid EV distributed
- Identical cross elasticities
- IIA property
- [TVR04]

Nested Logit Model (NL)

$$U_f = V_f + \boxed{\varepsilon_m + \varepsilon_{fm}} \quad \forall\, f \in C_m | m$$

- No constant substitution
- Correlation of pairs of f
- Generalization of MNL
- More realistic choice behaviour
- [WK01]

Application I: Modelling Demand - Fare Class Choice

Utility Function

Considered function of deterministic utility

$$U_f = \beta_{f,asc} + \beta_{price} \cdot x_{f,price} + \beta_{f,prp} \cdot x_{prp} + \beta_{f,gen} \cdot x_{gender} + \varepsilon_f \quad \forall\ f$$

- Generation of synthetic datasets based on U_f
- NL and MNL moddels differ according to ε_f
- Coefficient values are provided $\rightarrow$ "true" values

Results

- Two populations with 10.000 individuals each
- Choice behaviour according to NL and MNL models
- Datasets exhibit expected substitution patterns
- Assumption: NL population exhibits "true" behaviour

Alternatives, Variables & Nesting

- Choice set contains 4 fare class alternatives i:
  - Regular (1) and discount (2) fare in Business Class
  - Regular (3) and discount (4) fare in Economy Class
  - No choice/Outside Alternative (OA) (5)
- Provided coefficient values
  - $\beta_{price} = -0.0040$
  - $\beta_{1,prp} = 2.0$, $\beta_{2,prp} = 1.5$, $\beta_{3,prp} = 1.0$, $\beta_{4,prp} = 0.5$, $\beta_{5,prp} = 0$
  - $\beta_{1,gen} = 0.8$, $\beta_{2,gen} = 0.5$, $\beta_{3,gen} = 0.2$, $\beta_{4,gen} = -0.1$, $\beta_{5,gen} = 0$
- Assingnment of alternatives to nests
  - Nest 1 = {1,2}
  - Nest 2 = {3,4}
  - Nest 3 = {5}

Tree Structure

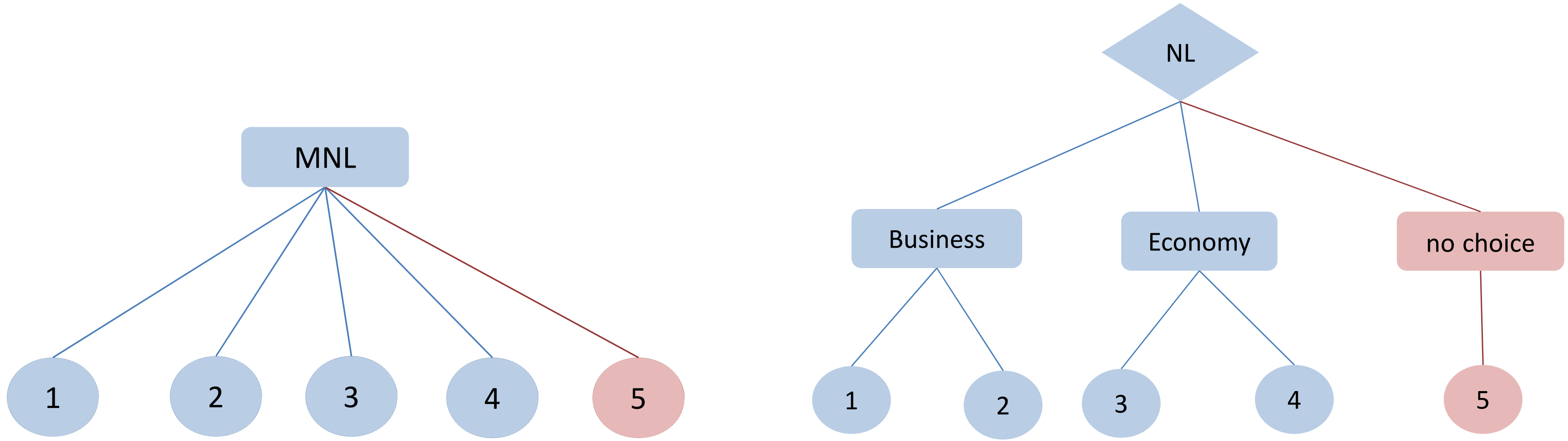


Figure: Tree structure for fare class choice in the MNL (left) and NL (right)

Demand Integration

1. Draw random sample of 2000 observations from **NL** population
2. Use sample as dataset for estimation
3. Estimate NL model from NL sample $\rightarrow$ NL coefficients
4. Estimate MNL model from NL sample $\rightarrow$ MNL coefficients
5. Put NL coefficients in optimization model with NL utility function
6. Put MNL coefficients in optimization model with MNL utility function
7. Compare results

Estimation Results

Parameter	Name	NL estimate	*t*-stat	MNL estimate	*t*-stat
1	$\beta_{asc,1}$	0.321	0.69	0.229	0.37
2	$\beta_{asc,2}$	1.83	4.40	2.51	8.23
3	$\beta_{asc,3}$	1.32	6.83	1.16	5.81
4	$\beta_{asc,4}$	1.98	14.31	2.15	17.44
5	β_{price}	-0.00459	-8.64	-0.00559	-15.75
6	$\beta_{prp,1}$	1.08	3.47	1.30	3.21
7	$\beta_{prp,2}$	0.361	1.95	0.347	1.80
8	$\beta_{prp,3}$	0.145	0.97	0.231	1.42
9	$\beta_{prp,4}$	-0.193	-1.60	-0.212	-1.74
10	$\beta_{gen,1}$	2.40	6.35	2.68	5.34
11	$\beta_{gen,2}$	1.47	7.53	1.45	7.16
12	$\beta_{gen,3}$	1.07	6.64	1.22	7.31
13	$\beta_{gen,4}$	0.516	4.14	0.477	3.80
14	NestA	1.73	1.77[1]	–	–
15	NestB	1.36	1.71[1]	–	–

[1] *t*-test against 1

Application II: Simulation based RM Optimization Model

Preface

Generate utility values for two populations of individuals

- Choice behavior based on MNL and NL models
- Maximum utility determines choice decision
- Linear model formulation
- General formulation in terms of applied DCM
- Calculation of optimal seat allocation per fare class
  - → Decision on fare classes offered to an individual
  - → Fare classes are offered in a revenue maximizing manner
  - → Choice sets vary across individuals
- Calculation of optimal price per fare class f
- Solved with Cplex in GAMS version 23.9

Definitions

Sets

S	Simulated sets of individuals
F	Set of fare classes indexed f, no choice/OA $f = 5$
N	Set of simulated individuals indexed n $\forall s \in S$

Parameters

$\bar{p}_f$	Upper bound for price variable in fare class f
L	Sufficiently large number
A	Sufficiently large number to guarantee $u_{snf} > 0$
C	Seat capacity of considered resource
d_{snf}	Deterministic utility plus A, without $\beta_{price} \cdot p_f$
β_{price}	Price coefficient

Definitions

Variables

u_{snf}	Utility individual n receives from choosing fare class f in s
$\tilde{u}_{sn}$	Maximum utility value for individual n in s
p_f	Price for a ticket in fare class f in s
π_{snf}	Price individual n pays for a ticket in fare class f in s
b_f	Maximum amount of bookings in fare class f in s
y_{snf}	$= 1$, if individual n chooses fare class f in s (0, else)
x_{snf}	$= 1$, if f is offered to n in s (0, else)
Z	Objective value

Objective Function

$$\max Z = \frac{1}{|S|} \cdot \sum_{s} \sum_{n} \sum_{f} \pi_{snf} \tag{1}$$

Price & Utility

$$u_{snf} - d_{snf} - \beta_{price} \cdot p_f \leq 0 \qquad \forall\ s, n, f \tag{2}$$

$$u_{snf} - d_{snf} - \beta_{price} \cdot p_f + L \cdot (1 - x_{snf}) \geq 0 \qquad \forall\ s, n, f \tag{3}$$

$$\pi_{snf} - \bar{p}_f \cdot y_{snf} \leq 0 \qquad \forall\ s, n, f \tag{4}$$

$$\pi_{snf} - p_f \leq 0 \qquad \forall\ s, n, f \tag{5}$$

$$\tilde{u}_{sn} - u_{snf} \geq 0 \qquad \forall\ s, n, f \tag{6}$$

$$u_{snf} - L \cdot x_{snf} \leq 0 \qquad \forall\ s, n, f \tag{7}$$

$$\tilde{u}_{sn} - u_{snf} - L \cdot (1 - y_{snf}) \leq 0 \qquad \forall\ s, n, f \tag{8}$$

Remaining Constraints

$$\sum_n y_{snf} - b_f \leq 0 \qquad \forall \ s, f < 5 \qquad (9)$$

$$\sum_f y_{snf} = 1 \qquad \forall \ s, n \qquad (10)$$

$$y_{snf} - x_{snf} \leq 0 \qquad \forall \ s, n, f \qquad (11)$$

$$x_{snf} - x_{sn-1f} \leq 0 \qquad \forall \ s, n > 1, f < 5 \qquad (12)$$

$$b_f \geq 0 \qquad \forall \ f \qquad (13)$$

$$p_f \geq 0 \qquad \forall \ f \qquad (14)$$

$$\pi_{snf} \geq 0 \qquad \forall \ s, n, f \qquad (15)$$

$$x_{snf} \in \{0, 1\} \qquad \forall \ s, n, f \qquad (16)$$

$$y_{snf} \in \{0, 1\} \qquad \forall \ s, n, f \qquad (17)$$

Results

Overview

- Results for varying price coefficient β_p
- β_p changes according to start value (MNL/NL) by +0.0005

$\Rightarrow$ Change in individuals price sensitivity

- Demand is constant with 10 generated individuals per simulation run
- Available Capacity is $C = 5$
- Simulated sets of individuals $S = 20$
- Comparison for NL and MNL models

Open fare classes for number of individuals

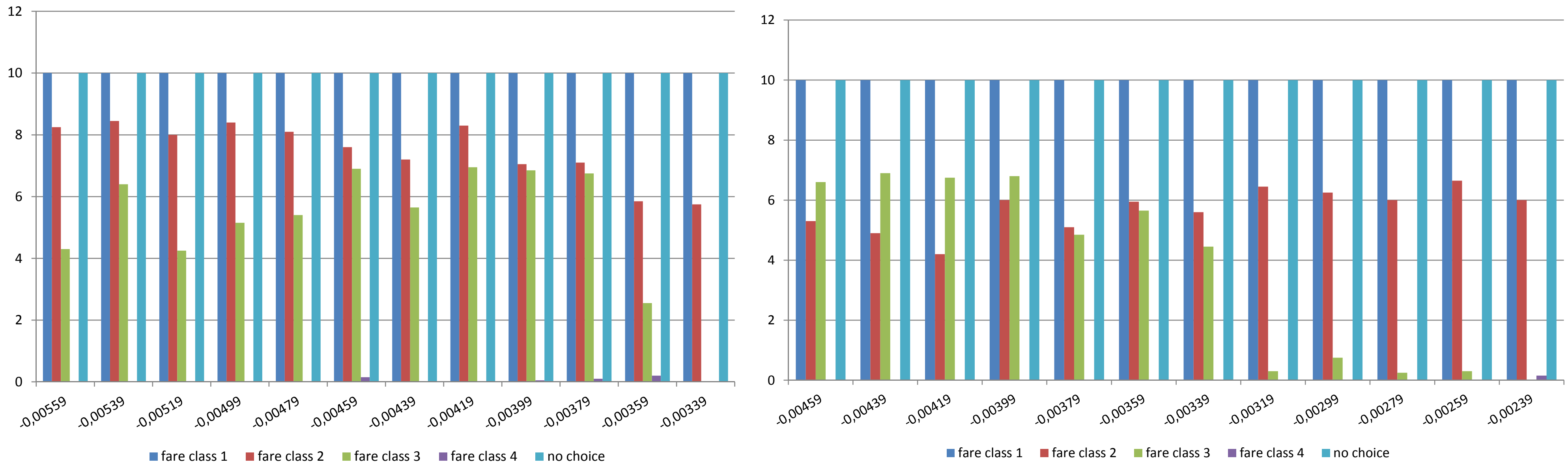


Figure: Open fare classes for MNL (left) and NL (right) utilities

Chosen fare classes on average

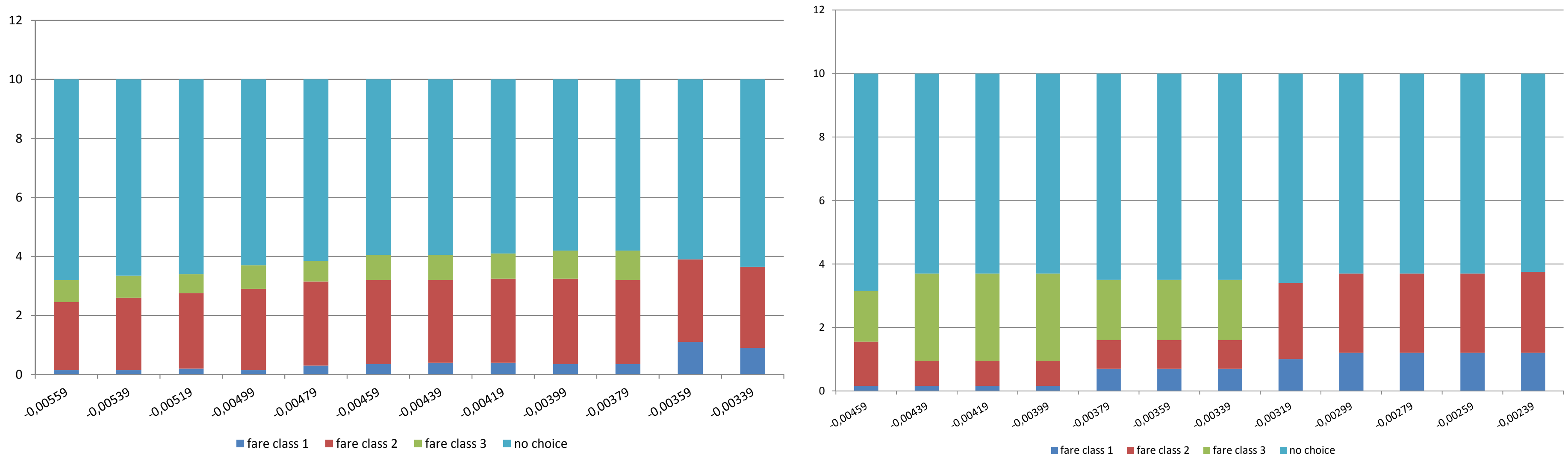


Figure: Number of bookings for MNL (left) and NL (right) utilities

Prices payed on average

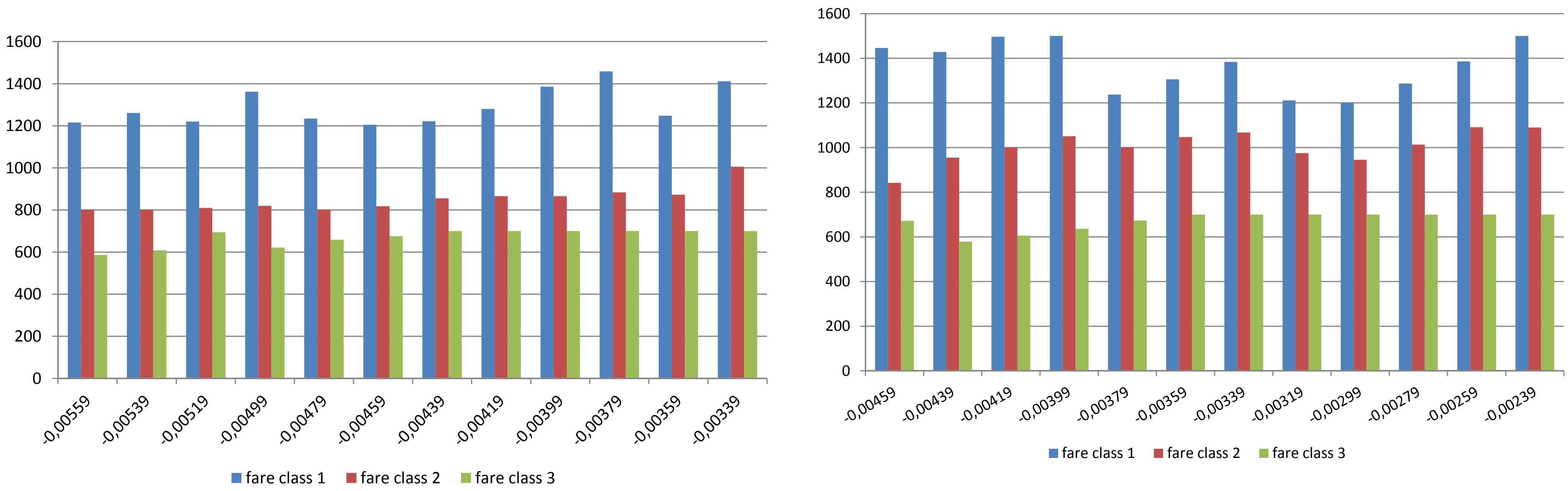
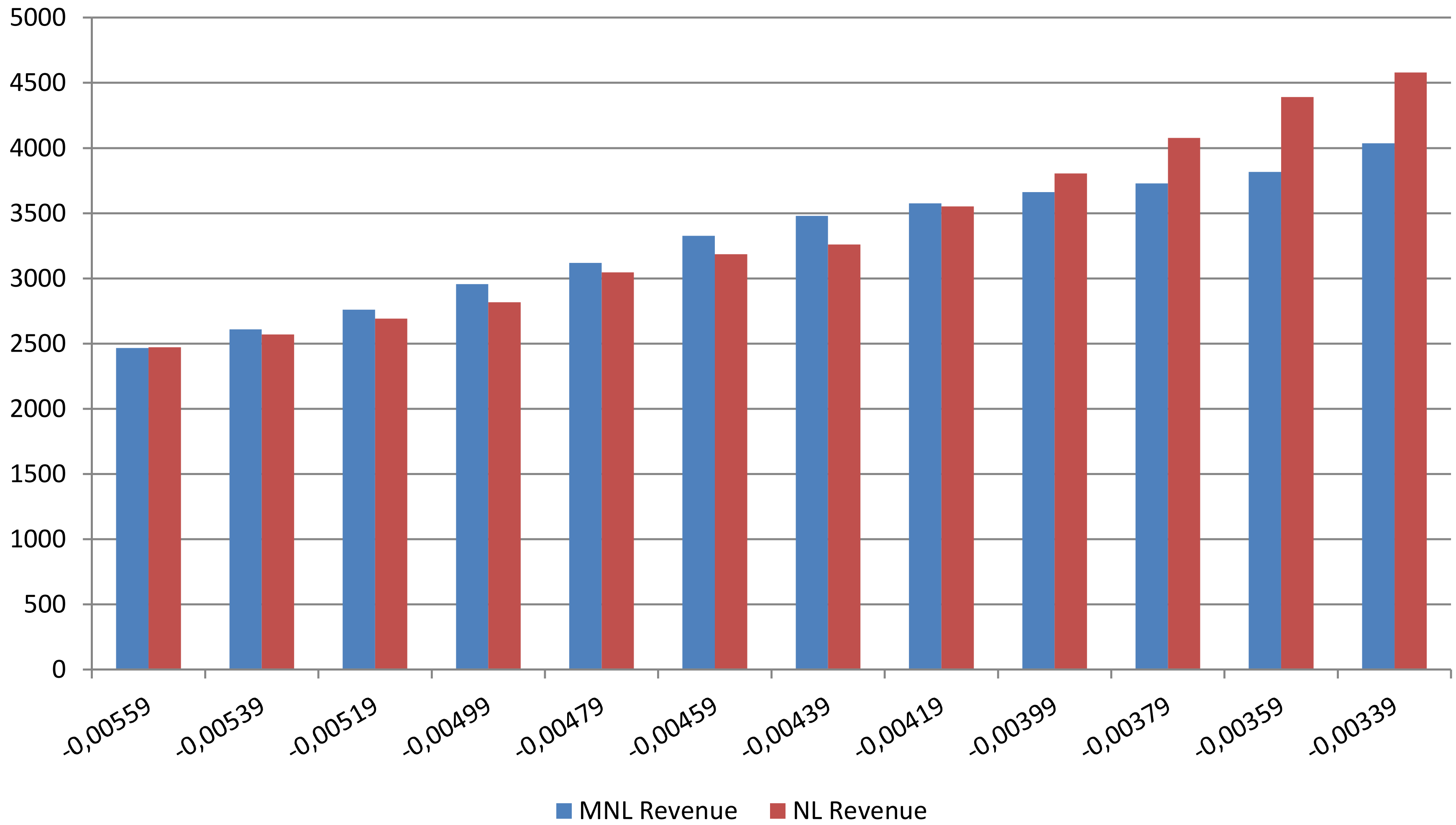
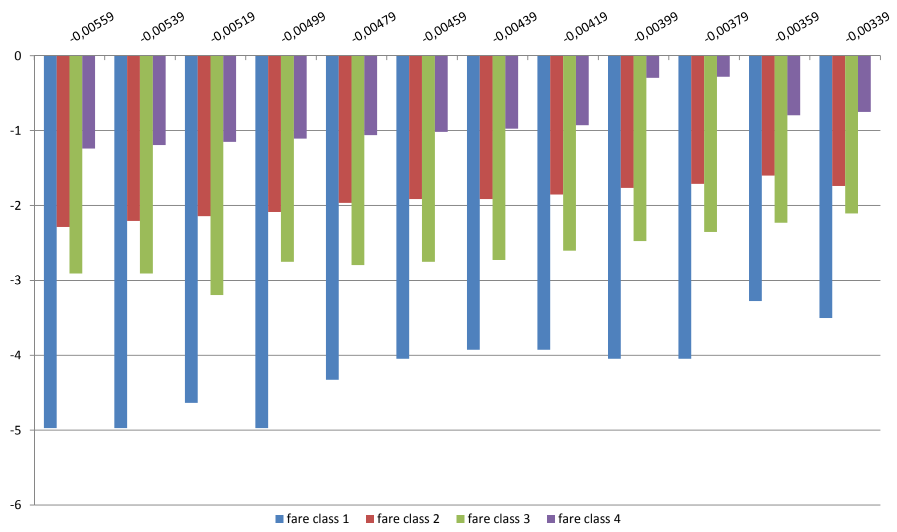


Figure: Prices payed on average for MNL (left) and NL (right) utilities

Average Revenues



Direct Elasticities



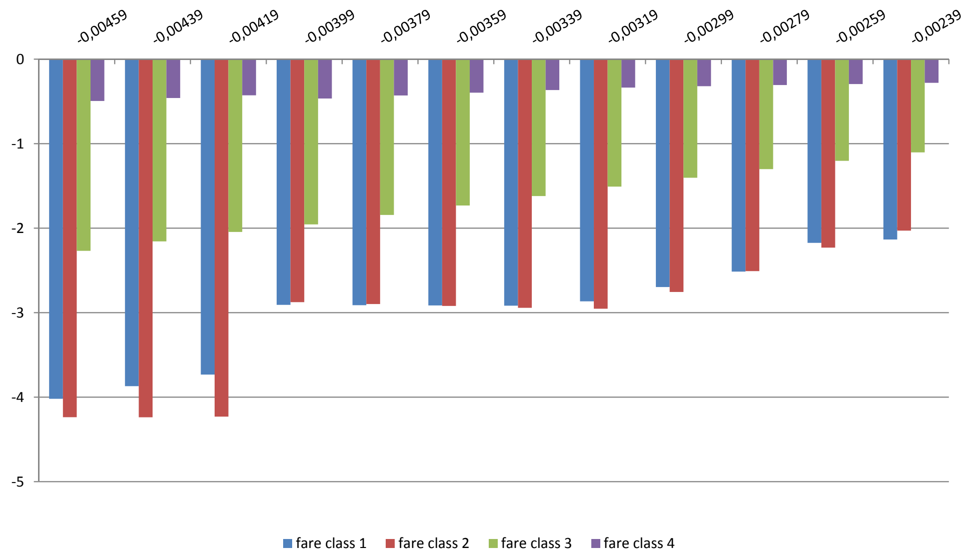


Figure: Direct elasticities for MNL (left) and NL (right) utilities

Cross Elasticities

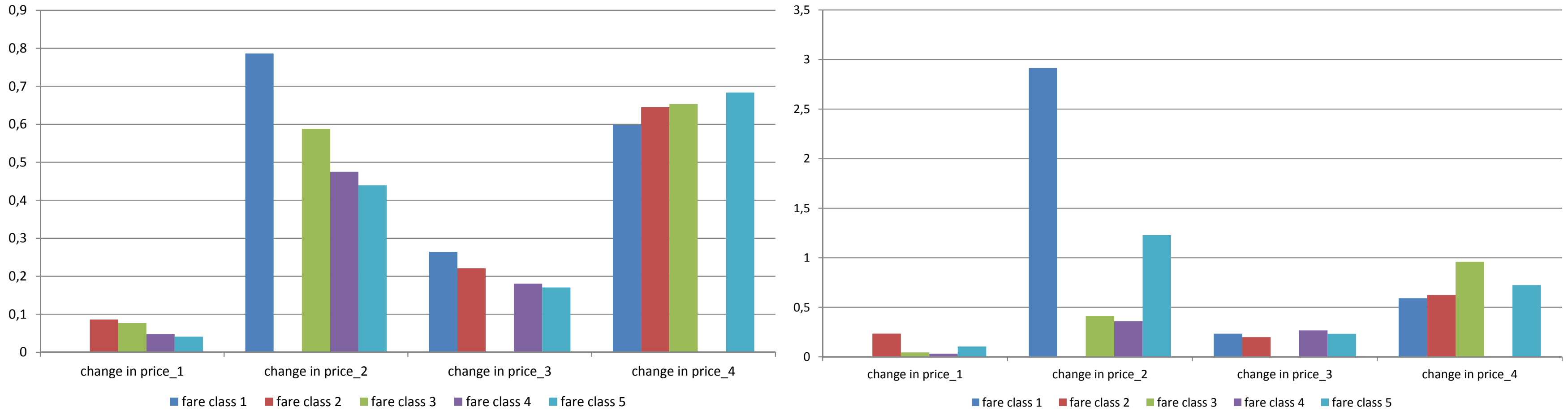


Figure: Cross elasticities for MNL (left) and NL (right) utilities

Thank you very much for your attention!

Coefficients

Coefficient		MNL		NL	
	true value	estimate	std	estimate	std
$\beta_{asc,1}$	0.50	0.869	0.193	0.388	0.194
$\beta_{asc,2}$	1.50	1.590	0.137	1.290	0.179
$\beta_{asc,3}$	1.50	1.510	0.0797	1.430	0.0671
$\beta_{asc,4}$	2.00	1.970	0.0555	1.890	0.0616
$\beta_{asc,5}$	0.00	.	.	.	.
β_{price}	-0.0040	-0.0042	0.000147	-0.0040	0.000226
$\beta_{purpose,1}$	2.00	1.960	0.150	1.970	0.142
$\beta_{purpose,2}$	1.50	1.640	0.0971	1.490	0.0853
$\beta_{purpose,3}$	1.00	1.130	0.0725	0.943	0.0652
$\beta_{purpose,4}$	0.50	0.600	0.0642	0.501	0.0548
$\beta_{purpose,5}$	0.00	.	.	.	.
$\beta_{gender,1}$	0.80	0.726	0.135	0.780	0.123
$\beta_{gender,2}$	0.50	0.494	0.0916	0.597	0.0822
$\beta_{gender,3}$	0.20	0.226	0.0701	0.193	0.0614
$\beta_{gender,4}$	-0.10	-0.0517	0.0614	-0.107	0.0530
$\beta_{gender,5}$	0.00	.	.	.	.
μ_1	1.9	.	.	1.760	0.169
μ_2	2.10	.	.	2.120	0.146
μ_3	1.00	.	.	1.00	.

Table: True coefficient values for the dataset generation of MNL and NL

Substitution Patterns I

MNL

Fare class	1	2	3	4	5
Market share	5.10	10.50	45.30	31.00	8.10

- Market shares when all fare classes are available

Fare class	1	2	3	4	5
Market share	6.96	14.97	65.56	0.00	12.51

- Alternative 4 is no longer available
- Market shares of 1, 2, 3 and 5 rise proportionally
- Ratio of substitution between any pair of alternatives is constant

$\rightarrow$ **Example:** ratio of alternatives $\frac{1}{3} = \frac{5.10}{45.30} = \frac{6.96}{65.56} = 0.11$

Substitution Patterns II

NL

Fare class	1	2	3	4	5
Market share	2.93	6.67	40.41	28.29	21.70

- Nest 1 = {1,2}, Nest 2 ={3,4}, Nest 3 ={5}

Fare class	1	2	3	4	5
Market share	3.53	8.26	60.03	0.00	28.18

- Market share of alternative in the same nest (=3) rises proportionally
- Flexible substitution *across* nests, constant substitution *within* nests

→ **Example:** ratio of alternatives $\frac{1}{3} = \frac{2.93}{40.41} = 0.073 \neq \frac{3.53}{60.03} = 0.059$

→ **Example:** ratio of alternatives $\frac{1}{2} = \frac{2.93}{6.67} = \frac{3.53}{8.26} = 0.43$

References

- Bitran, G. and R. Caldentey: An overview of pricing models for revenue management. Manufacturing & Service Operations Management, 5(3):203–229, 2003.
- Talluri, K. and G. Van Ryzin: Revenue management under a general discrete choice model of consumer behavior. Management Science, 50(1):15–33, 2004.
- Wen, C.H. and F.S. Koppelman: The generalized nested logit model. Transportation Research Part B: Methodological, 35(7):627–641, 2001.