Preventive Health Care Facility Location Planning with Quality–Conscious Clients

(Working Paper)

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The objective of probabilistic choice models for locating preventive health care facilities is the maximization of the expected participation in a screening program for, e.g., early detection of breast cancer in women. In contrast to sick people who need urgent medical attention, the clients of preventive health care choose whether to go for a certain facility location or not to take part in the program. In prevailing scientific papers it is assumed that waiting time for an appointment and the quality of care do not influence clients’ choice behavior. Therefore, the decision is only about the facilities’ locations and the number of servers per facility. However, it has been shown that this assumption yields suboptimal results in terms of participation.

In this contribution we consider clients’ utility function to include variables denoting the waiting time for an appointment and the quality of care. Both variables are defined as functions of a facility’s utilization. At first glance, this yields a mixed integer non–linear model formulation. As commonly modeled in empirical choice studies, we assume that the waiting time for an appointment can be considered to be categorial, i.e. the variable takes on only a few discrete values. The minimum quantity requirement (as a proxy for quality of care) is considered as a categorial variable as well, i.e., it indicates whether a facility satisfies the requirement or not. These assumptions allow us to employ a segmentation approach to formulate a mixed integer linear program. We show that the problem can be solved to optimality in acceptable time, applying GAMS / CPLEX to our instances, based on both artificial data as well as in the context of a case study based on empirical data.

1 Introduction

Preventive health care is beneficial to society because it enables the early detection of an illness and helps to interrupt its development. This is less costly and harmful than curing diseases at advanced stages. Established preventive health care programs (e.g. breast or colorectal cancer screenings) help to increase the number of early detections. To save both lives
and potential medical treatment costs, policy decision makers aim at raising participation in those programs. Whereas patients have little influence on where they get medical help in urgent cases, preventive measures can be planned in advance and allow for patients’ choice. Empirical studies show that accessibility is a very important factor that drives people’s participation. Our objective is to develop a methodology for setting up a limited number of facilities such that patients’ participation in a preventive screening program is maximized.

In a substantial body of facility location literature it is assumed that clients’ choices only depend on the distances between the locations of residence and those of medical facilities, or the corresponding travel time, respectively. It has been proven that proximity and travel time indeed have a strong impact on those decisions (Train 2009). Common models locate service facilities subject to minimizing the travel distances and allocate them to demand nodes. This neglects that patients are also interested in many more attributes, as other studies illustrate, and that they do not necessarily choose an alternative deterministically. A probabilistic approach allows for more uncertainty due to the decision maker’s lack of information (Koppelman & Bhat 2006). There is evidence that patients are also sensitive to the practitioner’s technical experience (quality of care) and waiting times for an appointment for an examination. However, the considerations of both aspects are at most implemented as capacity constraints so far. In comparison to that, the enhancements of our novel model are mainly the following. First, both quality and waiting time of established facilities are no longer fixed via hard constraints, but can occur in more or less attractive states. Second, we incorporate that those properties do have an impact on demand, as attractiveness varies. Quality and waiting time are moreover a result of the number of patients that access service and therefore they are dependent upon demand in turn. The resulting endogeneities and feedbacks are mostly managed by queueing theory approaches, whereas we integrate these issues into a deterministic mixed integer linear problem via common discretization.

So far, Zhang et al. (2009) and Zhang et al. (2010) provide non–linear location–allocation models with respect to congestion in the context of preventive health care network designs. Verter & Lapierre (2002) present a preventive health care facility location problem with an emphasis on distance and implement a minimum quantity requirement to ensure quality. Population centers are allocated to exactly one established facility each. Vidyarthi & Kuzgunkaya (2014) investigate the trade–off between the waiting and travel time patients are confronted with while simultaneously determining both the facilities’ locations and their capacities as well as the allocation of clients to them. The first and non–linear model formulation, which forms the basis of this paper, borrows parts from the models of Wang et al. (2002), Benati & Hansen (2002), Marianov et al. (2008). The linear model of major interest within the subsequent sections is essentially derived from Haase & Müller (2015). The final step of implementing a linear reformulation of originally non–linear expressions within the model follows ideas of Haase (2009), Aros-Vera et al. (2013), Haase & Müller (2015). Nonetheless, all cited past work covers only related problems and constitutes no solution method for our problem formulation.

2 Theoretical Framework

Discrete choice models are a tool to both analyze and predict individual choice behavior. An individual chooses exactly one alternative from a finite set of available, mutually exclusive, and collectively exhaustive alternatives. In our case the choice set includes all available
facilities and the alternative of not choosing any facility, i.e., the “no–choice” alternative. An individual’s utility for an alternative is a result of the alternative’s attributes as well as the individual’s characteristics. Following the utility maximization choice rule an individual chooses the available alternative that dominates all other alternatives by having the highest utility (Koppelman & Bhat 2006). That is, individual \( n \in N \) chooses alternative \( j \in J \), iff

\[
U_{n,j} > U_{n,h} \quad \forall \ h \in J \land h \neq j,
\]

(1)

where \( U_{n,j} \) is individual \( n \)’s perceived utility value for alternative \( j \). Since the decision making process is mostly not completely understood by the analyst, probabilistic choice models are used to account for unobserved characteristics and incomplete information. For that reason the utility is decomposed into two components:

\[
U_{n,j} = v_{n,j} + \epsilon_{n,j} \quad \forall \ n \in N; \ j \in J,
\]

(2)

where \( v_{n,j} \) denotes the deterministic utility part for an individual \( n \) and alternative \( j \), which is observed by the analyst. \( \epsilon_{n,j} \) is a stochastic error term that equals the difference between the known deterministic utility and the utility \( U_{n,j} \) used by the individual, which is generally unknown.

Since \( U_{n,j} \) is a random quantity, the analyst can only make probability statements about (1). The probability \( p_{n,j} \) that individual \( n \) chooses alternative \( j \) is

\[
p_{n,j} = P( U_{n,j} > U_{n,h} \quad \forall \ h \in J \land h \neq j ) .
\]

(3)

The multinomial logit model (MNL) (McFadden 1974) is based on the assumption that the error components \( \epsilon_{n,j} \) are independent and identically type I extreme–value (also: Gumbel) distributed (iid EV) across alternatives as well as across individuals. Therefore, the logit choice probability

\[
p_{n,j} = \frac{e^{v_{n,j}}}{\sum_{h \in J} e^{v_{n,h}}} \quad \forall \ n \in N; \ j \in J
\]

(4)

can be derived from (3) (Train 2009). A fundamental property of the MNL is the so–called irrelevance of independent alternatives (IIA). Consider the ratio of any two alternatives’ choice probabilities (odds):

\[
\frac{p_{n,j}}{p_{n,h}} = \frac{e^{v_{n,j}}}{\sum_{k \in J} e^{v_{n,k}}} / \frac{e^{v_{n,h}}}{\sum_{k \in J} e^{v_{n,k}}} = \frac{e^{v_{n,j}}}{e^{v_{n,h}}} = e^{v_{n,j} - v_{n,h}} .
\]

(5)

This means that the ratio of choice probabilities of two alternatives does not depend on the probabilities for any other alternative. The most important advantage of this property is the implication that alternatives (here: facility locations) can be added to or removed from the choice set without giving rise to the structure of the MNL (Koppelman & Bhat 2006). For our purposes, the deterministic utility is mainly describable as a linear function of an individual’s travel time to a location, the facility’s quality of care, and waiting time for an appointment at that service center. Travel time is a constant and waiting time is dependent on other participants’ choice. Both will generally have a negative impact on utility. A high quality level can be accredited by an authority and is suspected to be positively rated
instead.

3 Model Development

Let there be $|\mathcal{I}|$ client groups that are identical concerning their observed characteristics. Clients choose whether to take part in a certain preventive health care program. Detailed points of interest may be the proximity to an out-patient service center, getting to know something about the center’s quality of treatment from by word of mouth or official ratings. The objective is to establish preventive health care facilities in a way that maximizes patients’ expected participation in the program, to indirectly reduce health risks and early detect illness.

On the one hand, both quality and waiting time are dependent on demand for health care service, measured as the number of arriving patients at service facilities. On the other hand, also the demand side is suspected to be dependent on supply, as quality and waiting time are part of the utility function. The challenge is to put this information with all its dependencies and feedbacks into a linear closed form. Therefore, our model formulations are based on the following two assumptions:

1. It is assumed that high quality of care is achieved if a health care service facility exceeds a certain number of carried out procedures, and it lacks it, if that number is not reached. This value might come from corresponding medical guidelines or case-based research. This assumption is justified by real existing minimum quantity requirements for some medical treatments and operations as well as by the findings of Schrag et al. (2000), Güneş et al. (2004), Harewood (2005), Levin et al. (2008), Zhang et al. (2012).

2. It is assumed that waiting times for an appointment correlate to the number of clients going for a facility (given its capacity). As more patients ask for appointments, waiting time will increase (Güneş et al. 2004, Viiala et al. 2007).

Both high examination quality and low waiting times for an appointment are desirable in view of patient satisfaction. We do not contemplate possible waiting times at the doctor’s surgery, since we consider a strategic planning horizon. Our model’s objective is to decide where preventive health care service facilities have to be located, whether they provide high quality as well as how long a patient has to wait for an appointment to maximize patients’ participation.

Inter alia Wang et al. (2002) and Marianov et al. (2008) developed mixed integer non-linear programs that account for probabilistic customer choice based on continuous waiting times. In contrast, we avoid the continuous nature of queueing theory, since it is difficult for clients to evaluate a difference between, e.g., 1 week and 1.1 weeks. Furthermore, assuming waiting time to be linear in utility is inconsistent with empirical knowledge (Koppelman & Bhat 2006). Thus, we carry out a discretization of waiting time. We thereby assume known facilities’ client demand thresholds that determine a change in the expected waiting time for an appointment faced by a patient. The resulting categories could be separated into e.g. “two weeks” and “four weeks”.

4
3.1 Non–linear Preventive Health Care Facility Location Planning Problem

An initial and probably most intuitive modeling approach leads to a non–linear formulation:

Sets and Indices

\[ I \quad \text{set of demand nodes, index } i \]
\[ J \quad \text{set of potential facility location nodes } J \subseteq I, \text{ indices } j \text{ and } k \]

Parameters

\[ L \quad \text{number of waiting time levels, index } l = 1, \ldots, L \]
\[ \mu_l \quad \text{demand threshold from which on waiting time level } l \text{ applies} \]
\[ \bar{\mu}_l \quad \text{demand threshold to which waiting time level } l \text{ applies} \]
\[ c_{i,j} \quad \text{constant part of deterministic utility function for demand node } i \text{ and facility at location } j \text{ with, for example, } c_{i,j} = \beta_{ASC} + \beta_D \cdot d_{i,j}, \text{ where } \beta_{ASC} \text{ is the alternative–specific constant for alternative } j, \beta_D \text{ the coefficient for distance and } d_{i,j} \text{ the distance from node } i \text{ to node } j \]
\[ v_{i,\text{no}} \quad \text{deterministic utility for demand node } i \text{ of not attending any facility ("no–choice" alternative), which is later on set to 0} \]
\[ \beta_Q \quad \text{utility coefficient for quality property (} \beta_Q > 0) \]
\[ \beta_W^l \quad \text{utility coefficient for waiting time level } l \text{ (} 0 > \beta_W^1 > \beta_W^2 > \ldots > \beta_W^L \text{)} \]
\[ g_i \quad \text{demand in node } i \text{ as the number of clients requiring health service} \]
\[ q_{\text{min}} \quad \text{minimum quantity requirement} \]
\[ r \quad \text{total number of available facilities} \]
\[ B \quad \text{sufficiently big number} \]

Variables

\[ Q_j = 1 \text{ if facility at location } j \text{ satisfies minimum quantity requirement; } 0 \text{ otherwise} \]
\[ W_{j,l} = 1 \text{ if waiting time level } l \text{ applies for facility at location } j; 0 \text{ otherwise} \]
\[ X_{i,j} = 1 \text{ if choice probability of clients } i \text{ going for facility at location } j \]
\[ Y_j = 1 \text{ if location } j \text{ is specified to offer health care service; } 0 \text{ otherwise} \]
\[ v_{i,j} = \text{deterministic utility of facility at location } j \text{ for demand node } i \]
\[ F = \text{objective function value (expected health care service participation)} \]

Maximize \[ F = \sum_{i \in I} g_i \cdot \sum_{j \in J} X_{i,j} \] (6)
subject to

\[ X_{i,j} = \frac{e^{v_{i,j}} \cdot Y_j}{\sum_{k \in J} e^{v_{i,k}} \cdot Y_k} \quad \forall i \in I; j \in J \quad (7) \]

\[ v_{i,j} = c_{i,j} + \beta^Q \cdot Q_j + \sum_{l=1}^{L} \beta^W_l \cdot W_{j,l} \quad \forall i \in I; j \in J \quad (8) \]

\[ \sum_{i \in I} g_i \cdot X_{i,j} \geq q_{\text{min}} \cdot Q_j \quad \forall j \in J \quad (9) \]

\[ \sum_{i \in I} g_i \cdot X_{i,j} \leq \mu_l + B \cdot (1 - W_{j,l}) \quad \forall j \in J; l = 1, \ldots, L \quad (10) \]

\[ \sum_{i \in I} g_i \cdot X_{i,j} \geq \mu_l \cdot W_{j,l} \quad \forall j \in J; l = 1, \ldots, L \quad (11) \]

\[ \sum_{l=1}^{L} W_{j,l} = Y_j \quad \forall j \in J \quad (12) \]

\[ \sum_{j \in J} Y_j = r \quad (13) \]

\[ v_{i,j} \geq 0 \quad \forall i \in I; j \in J \quad (14) \]

\[ X_{i,j} \geq 0 \quad \forall i \in I; j \in J \quad (15) \]

\[ W_{j,l} \in \{0; 1\} \quad \forall j \in J; l = 1, \ldots, L \quad (16) \]

\[ Q_j \in \{0; 1\} \quad \forall j \in J \quad (17) \]

\[ Y_j \in \{0; 1\} \quad \forall j \in J \quad (18) \]

The objective function (6) maximizes the expected participation measured as the number of patients that are expected to access preventive health care service. (7) are the MNL choice probabilities for an individual to access service at a certain facility. \( v_{i,\text{no}} \) denotes individuals’ utility of accessing no service at all ("no–choice" or "opt–out"), which is also an alternative out of the available choice set. By the link with the decision variable \( Y_j \), other alternatives are taken into consideration, if they are established. By (8) we calculate the deterministic utilities \( v_{i,j} \) with some constant \( \bar{c}_{i,j} \) in dependence of a facility’s weighted levels of quality and waiting time.

(9) ensure that a facility’s binary quality indicator variable is set to zero if the number of served patients (left hand side) is lower than the minimum quantity requirement \( q_{\text{min}} \). Instead, if this is exceeded, \( Q_j \) can get the value 1, which is realized in conjunction with (6). Basically, this constraint is also applied in Verter & Lapierre (2002), Zhang et al. (2012), Haase & Müller (2015), but here the parameter is linked with a binary variable, so it is also allowed to fall below the threshold, which has a negative impact on quality in return.

In an analogous manner a facility’s waiting time level is determined by (10) and (11). If the expected demand is greater than a threshold \( \bar{\mu}_l \), a sufficiently big value \( B \) has to be added on the right hand side to make (10) valid. This forces the corresponding waiting time level variable to become 0. Out of the set of variables \( W_{j,l} \) that can become 1 by those constraints, the one corresponding to the lowest feasible waiting time level, meaning highest utility, is used due to (11) (see Fig. 1). The utilization of (6) may be insufficient to determine the
correct level because of feedbacks with (9). Otherwise (11) would not be necessary. This kind of service level constraint is also been taken into account in Berman & Drezner (2006), Haase & Müller (2015), Marianov & Serra (2002), Wang et al. (2002), Zhang et al. (2012), but they only set an upper bound on capacity to limit the expected waiting time, whereas our model allows for more segmentation. (12) ensure that an established facility can only have exactly one waiting time level. (13) ensures that \( r \) servers are established. (14)–(18) define the variables’ domains.

3.2 Mixed Integer Linear Preventive Health Care Facility Location Planning Problem

It is desirable to transfer the non–linear model (6)–(18) into a linear form, as linear models are basically much easier to solve. A comparison of different linear reformulations for related problems provide Haase & Müller (2014). Our approach makes use of the findings of Haase (2009), Aros-Vera et al. (2013), Haase & Müller (2015) as well as the MNL’s IIA property: the basic idea is to provide choice probabilities, which can be calculated in advance, as input parameters and take advantage of their ratios, which are constant by applying the MNL.
We compress the representation of quality and waiting time levels in a more general “mode” structure (see Fig. 2). This also makes it easier to exclude infeasible combinations of quality and waiting time levels. Müller & Haase (2014) introduce a customer demand segmentation approach. Here, this idea is applied to the supply side, which brings a lot of necessary adaptions with it. Each combination of facility location and mode is considered as a separate choice alternative. For a short example, let there be one facility with two possible modes. This results in two alternatives of the choice set. Those two virtual facilities are dependent on each other. Particularly, they can not be established both simultaneously, because only exactly one mode can be assigned to a facility. Consider the additional

**Sets and Indices**
- $\mathcal{M}$ set of a facility’s modes it can be established in (quality and waiting time for an appointment), indices $m$ and $n$

**Parameters**
- $l_m$ lower interval threshold for mode $m$, measured in number of clients
- $\bar{l}_m$ upper interval threshold for mode $m$, measured in number of clients
- $v_{i,j,m}$ deterministic utility of clients located at $i$ going for a facility located at $j$ and being in mode $m$
- $p_{i,j,m}$ choice probability of clients at node $i$ who access service at a facility located at $j$ being in mode $m$ given that all facilities and all modes $(k,n) \in \mathcal{J} \times \mathcal{M}$ are established, i.e., $p_{i,j,m} = e^{v_{i,j,m}} \sum_{k \in \mathcal{J}} \sum_{n \in \mathcal{M}} e^{v_{i,k,n}}$
- $p_{i,\text{no}}$ choice probability of clients at node $i$ of accessing no service given that all potential facilities and all modes $(j,m) \in \mathcal{J} \times \mathcal{M}$ are established, i.e., $p_{i,\text{no}} = e^{v_{i,\text{no}}} \sum_{j \in \mathcal{J}} \sum_{m \in \mathcal{M}} e^{v_{i,j,m}} = 1 - \sum_{j \in \mathcal{J}} \sum_{m \in \mathcal{M}} p_{i,j,m}$
- $p_{i,j,m}^{\text{max}}$ maximum choice probability of clients at node $i$ who access service at a facility located at $j$ being in mode $m$ given that $(j,m)$ is the only facility established, i.e., $p_{i,j,m}^{\text{max}} = e^{v_{i,j,m}} / e^{v_{i,\text{no}}} + e^{v_{i,j,m}}$

**Variables**
- $Z_i$ cumulative choice probability of clients at node $i$ of not accessing any facility (“no-choice”)
- $X_{i,j,m}$ choice probability of clients at node $i$ who access service at a facility located at $j$ being in mode $m$
- $Y_{j,m} = 1$ if location $j$ is specified to offer health care service in mode $m$; 0 otherwise

Maximize $F = \sum_{i \in \mathcal{I}} g_i \cdot \sum_{j \in \mathcal{J}} \sum_{m \in \mathcal{M}} X_{i,j,m}$ (19)
subject to

\[ Z_i + \sum_{j \in \mathcal{J}} \sum_{m \in \mathcal{M}} X_{i,j,m} = 1 \quad \forall \ i \in \mathcal{I} \]  
\[ X_{i,j,m} \leq p_{i,j,m}^{\text{max}} \cdot Y_{j,m} \quad \forall \ i \in \mathcal{I}; j \in \mathcal{J}; m \in \mathcal{M} \]  
\[ X_{i,j,m} \leq \frac{p_{i,j,m}}{p_{i}^{\text{no}}} \cdot Z_i \quad \forall \ i \in \mathcal{I}; j \in \mathcal{J}; m \in \mathcal{M} \]  
\[ X_{i,j,m} \geq \frac{p_{i,j,m}}{p_{i}^{\text{no}}} \cdot Z_i + Y_{j,m} - 1 \quad \forall \ i \in \mathcal{I}; j \in \mathcal{J}; m \in \mathcal{M} \]  
\[ \sum_{i \in \mathcal{I}} g_i \cdot X_{i,j,m} \geq \underline{l}_m \cdot Y_{j,m} \quad \forall \ j \in \mathcal{J}; m \in \mathcal{M} \]  
\[ \sum_{i \in \mathcal{I}} g_i \cdot X_{i,j,m} \leq \bar{l}_m \cdot Y_{j,m} \quad \forall \ j \in \mathcal{J}; m \in \mathcal{M} \]  
\[ \sum_{m \in \mathcal{M}} Y_{j,m} \leq 1 \quad \forall \ j \in \mathcal{J} \]  
\[ \sum_{j \in \mathcal{J}} \sum_{m \in \mathcal{M}} Y_{j,m} = r \]  
\[ X_{i,j,m} \geq 0 \quad \forall \ i \in \mathcal{I}; j \in \mathcal{J}; m \in \mathcal{M} \]  
\[ Z_i \geq 0 \quad \forall \ i \in \mathcal{I} \]  
\[ Y_{j,m} \in \{0; 1\} \quad \forall \ j \in \mathcal{J}; m \in \mathcal{M} \]

The objective function (19) maximizes the participation (measured as the number of patients that are expected to access preventive health care service). According to Haase & Müller (2015), (20)–(23) together with (19) are a linear reformulation of (7). (20) ensure that a demand node’s final choice probabilities for going for service facilities as well as non–attendance sum up to 1. The linking constraints (21) allow choice probabilities for a facility to be greater than 0 only if the corresponding facility is actually established. Allowing for \( p_{i,j,m}^{\text{max}} \) yields to a smaller upper bound by the corresponding LP–relaxation than just using \( X_{i,j,m} \leq Y_{j,m} \) and to tighter bounds for \( X_{i,j,m} \) (Haase & Müller 2015), because \( p_{i,j,m}^{\text{max}} \) is distinctly smaller than 1.

(22) and (23) ensure that the pre–calculated constant substitution ratios between the choice probabilities for any two alternatives are obeyed. Note that both \( X_{i,j,m} \) and \( Z_i \) have to be adjusted by the solver to fulfill that requirement. One could find the alternative formulation \( \frac{X_{i,j,m}}{Z_i} = \frac{p_{i,j,m}}{p_{i}^{\text{no}}} \) more intuitive. Computational experiments without (23) sometimes led to wrong probability values.

The correct mode in which a facility is established in is selected by (24) (lower mode interval threshold) and (25) (upper threshold). Both restriction blocks are necessary because there possibly is no clear utility ascent or descent with progressing modes so that one can not exploit the optimization direction. If a certain facility is established the left hand side has to be between the lower and the upper threshold. Probably, the most complicated feature in this model is that otherwise the choice probabilities are zero anyway. In the non–linear formulation, the left hand side always stays the same for one facility and its mode can be selected on the right hand side. Within this model, demand is only positive for exactly one mode, if that facility is established at all. For all other modes demand is 0, because they do not even exist. (26) ensure that a facility can either only be established in exactly one mode
or not at all and (27) ensures that \( r \) servers are established. (28)–(30) define the variables’ domains.

References


